Solutions to Quiz 6

1. Let $f(x) = (0.75)^x$.
   
   (a) Estimate $f'(0)$ using $\Delta x = 0.001$.
   
   Solution: 
   $$f(0.001) - f(0) \approx -0.28764,$$
   and 
   $$f(0) - f(-0.001) = 0.001 \approx -0.29772.$$ 
   Thus, it seems that $f'(0) \approx -0.297$.

   (b) Explain why the answer you found in (a) has the sign it does. (Positive or negative.)
   
   Solution: Since $f$ is decreasing everywhere, its derivative should be negative everywhere.

   (c) What is $\ln(0.75)$? What do you notice?
   
   Solution: $\ln(0.75) \approx -0.287682$, which is just about the value of $f'(0)$!

2. On the graph below, indicate in which regions $f'(x) > 0$, in which regions $f'(x) < 0$, and at what points $f'(x) = 0$.

   Solution: $f$ is decreasing for $x$ between $-1$ and 2 and for $x > 4$. On these regions, $f'(x) < 0$. $f$ is increasing for $x < -1$ and $x$ between 2 and 4, so $f'(x) > 0$ on these regions. Finally, since the tangent lines at $x = -1, 2, 4$ are horizontal, $f'(-1) = f'(2) = f'(4) = 0$. 

   ![Graph of function $f(x) = (0.75)^x$ showing regions where $f'(x)$ is positive, negative, or zero.]}