3. (a) \( \lim_{x \to -3} f(x) = \infty \) means that as \( x \) gets closer to \(-3\) from either side, \( f(x) \) gets larger and larger, increasing without bound.

(b) \( \lim_{x \to 4} f(x) = \infty \) means that as \( x \) gets closer to \(-3\) from the right, the graph of \( f \) goes down farther and farther, decreasing without bound. It gives us no information as to what is happening to the left of 4.

4. (a) This limit is 2 since the graph passes smoothly through \((0, 2)\).

(b) This limit is 4 since the graph rises up to 4 from the left and we ignore what is happening to the right of \( x = 2 \).

(c) This limit is 2 since the \( y \)-values on the graph approach 2 if we come in from the right, and we ignore what is happening to the left of \( x = 3 \).

(d) This limit does not exist since the left- and right-hand limits do not agree.

(e) \( f(3) = 3 \).

6. (i) The limit from the left does not exist since there is no single value that the function approaches as \( x \) nears 5 from the left; instead, it appears to range infinitely often between 2 and 4.

8. (a) \( \lim_{x \to 2} R(x) = -\infty \) since the left- and right-hand sides both drop down without bound near \( x = 2 \). Note that the limit does not exist; we are just specifying the manner in which it fails to exist.

(b) \( \lim_{x \to 5} R(x) = \infty \); see (a).

(c) \( \lim_{x \to -3^{-}} R(x) = -\infty \).

(d) \( \lim_{x \to -3^{+}} R(x) = \infty \).

(e) The vertical asymptotes are \( x = -3, x = 2, \) and \( x = 5 \).

18. I will just do the closest ones. \(-0.999 : \frac{(-0.999)^2 - 2(-0.999)}{(-0.999)^2 - (-0.999) - 2} = -999\). Coupled with the others you calculated, it looks like this is going to \(-\infty\). \(-1.001 : \frac{(-1.001)^2 - 2(-1.001)}{(-1.001)^2 - (-1.001) - 2} = 1001\). This appears to be approaching \( \infty \). The limit does not exist.

21. I will try \( x = 0.001 \) and \( x = -0.001 \) and look at a graph. \( x = 0.001 : \frac{\sqrt{0.001 + 4} - 2}{0.001} \approx 0.249984 \). \( x = -0.001 : \frac{\sqrt{-0.001 + 4} - 2}{-0.001} \approx 0.250016 \). It looks like the limit is \( \frac{1}{4} \), and the graph seems to agree with this.
25. If $x > -3$ (as is the case when $x \to -3^+$), then $x + 3$ is positive, and so is $\frac{x + 2}{x + 3}$. Therefore, 
\[
\lim_{x \to -3^+} \frac{x + 2}{x + 3} = \infty.
\]

40. If $v < c$, then $\frac{v^2}{c^2} < 1$. As $v \to c^-$, however, $\frac{v^2}{c^2} \to 1^-$. Thus, the denominator is going to zero, but the numerator is not. This causes an infinite limit. Since the numerator and denominator are both positive, the limit is $\infty$. Physically, this means that as we try to accelerate the particle to the speed of light, its mass has to grow without bound. That’s silly! (We think...)