

MTH 142

Final Exam

Thursday, May 6, 2004

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Remember to **show your work**. Unsupported solutions will receive no credit.

1. (16 points) Integrate using the techniques we have studied. You may check your work with the table of integrals, but you must show me that you know how to integrate without them, too.

(a) $\int x^2 \cos x^3 dx$

(b) $\int x e^{-2x} dx$

(c) $\int \sqrt{1-x^2} dx$

2. (4 points) Evaluate the integral $\int_1^{\infty} \frac{1}{(2x+1)^2} dx$, if possible.

NOTE: For the rest of the problems, you may use a table of integrals to integrate.

3. (12 points) Consider $I = \int_0^{\sqrt{\pi}} \sin(x^2) dx$.

(a) Estimate I using the Trapezoid Rule and $n = 4$.

(b) Estimate I using Simpson's Rule and $n = 4$.

(c) Evaluate I on your calculator and compare to your answers in (a) and (b).

4. (4 points) Let R be the region bounded by the graphs of $y = x$ and $y = \sqrt{x}$. Determine the volume of the solid formed by revolving R about the line $x = 1$.

5. (4 points) Determine the arc length of the curve $y = x^2$ from $x = 0$ to $x = 3$.

6. (4 points) Find the center of mass of the region bounded by the x -axis and the graph of $y = -x^2$ between $x = 0$ and $x = 2$.

7. (12 points) Determine whether the given sequence converges or diverges. If a sequence converges, find its limit.

(a) $a_n = \frac{3n+1}{5n+6}$.

(b) $a_n = \frac{1}{n}$

(c) $\frac{(-1)^n n}{n^2+1}$

8. (16 points) Determine which series converge and which diverge. If a series converges, find its sum.

(a) $\sum_{n=1}^{\infty} \frac{2}{n^2+3n}$

(b) $\sum_{n=1}^{\infty} \frac{4}{n}$

(c) $\sum_{n=1}^{\infty} 3 \cdot \left(\frac{3}{5}\right)^{n-1}$

(d) $\sum_{n=1}^{\infty} 3(1.001)^{n-1}$

9. (4 points) Convert $2.51515151\dots$ into a fraction.

10. (12 points) Consider the polar equation $r = 1 - \cos 2\theta$.

(a) Sketch a polar graph of the points satisfying the given polar equation.

(b) Find the slope of the tangent line to the graph at $\theta = \frac{\pi}{6}$.

(c) Find the area of the region enclosed by the graph.

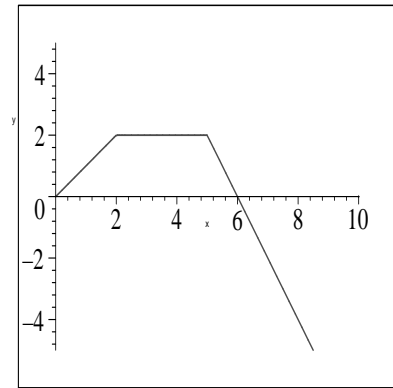
11. (4 points) If $F(x) = \int_1^{\sqrt{x}} \frac{\sqrt{1-t^2}}{\sqrt{1+t^2}}$, what is $F'(x)$?

12. (4 points) Wellbutrin is a drug used in the treatment of depression and also to treat various addictions (including nicotine). After one day, about 45.29% of the previous day's dose remains in a patient's system. If each dose is 150 mg, how much Wellbutrin is in a patient's system immediately after a dose in the long term? [Hint: You will need to set up a geometric series.]

13. (4 points) Define $F(x)$ by $F(x) = \int_0^x f(t)dt$, where $y = f(t)$ is the graph shown.

(a) What is the numerical value of $F(4)$?

(b) What is $F'(8)$?



14. (10 points) BONUS!! State and prove the Mean Value Theorem for Integrals.