Review for Final Exam

MATH 249

The following is offered without warranty, expressed or implied. You are responsible for all material covered.

Logistics

1. Sage will be available, but you may not access the internet otherwise.
2. No cell phone calculators, iPads, etc.
3. You may use your blue book after you hand in the first page.
4. You will need to state Green’s Theorem, Stokes’ Theorem, and the Divergence Theorem without the assistance of your blue book.
5. The exam will include roughly half new material and half old material.

Content

1. 3-D coordinates and vectors.
   (a) Dot product, cross product, and their properties and uses
2. Equations of lines and planes
3. Arc length, curvature, $T, N, B$
4. Multivariable functions
   (a) Graphs
   (b) Partial derivatives
   (c) Equations of tangent planes
   (d) Linearization and differentials
   (e) The chain rule
   (f) Directional derivatives
   (g) The gradient
      i. Used to compute directional derivatives
      ii. Normal to level curves/surfaces
      iii. Direction of greatest increase
   (h) Optimization
      i. Local extrema ($D$)
      ii. Absolute extrema (Critical points and boundary)
      iii. Lagrange multipliers
5. Multiple integration
   (a) Double integrals
      i. Definition/concept
      ii. Calculation (e.g., midpoint rule)
   (b) Iterated integrals
      i. Definition/concept
      ii. Fubini’s Theorem
      iii. General regions
      iv. Polar coordinates ($dA = r dr d\theta$)
      v. Computing area with a double integral
      vi. General changes of variable
   (c) Triple integrals
6. Vector Fields
   (a) Definition/concept
   (b) Gradient vector fields/conservative vector fields (and potential functions)
   (c) Line/path integrals of scalar functions and vector fields (including meaning and interpretation)
   (d) Fundamental Theorem for line integrals
   (e) Independence of path
   (f) Green’s Theorem
   (g) Curl and divergence
   (h) Test for conservatism

7. Parametric surfaces
   (a) Surface area
   (b) Tangent planes

8. Surface integrals
   (a) Scalar functions – definition and computation (parametric and \( z = g(x, y) \), what is \( dS \)?)
   (b) Flux integrals – definition and computation (parametric and \( z = g(x, y) \), what is \( d\vec{S} \)?)
   (c) Stokes’ Theorem
   (d) Divergence Theorem

**Advice**

(a) Sketch the domain of integration for every integration problem.

(b) Remember to include \( dA \) and \( dV \) as appropriate to the coordinate system you choose.

(c) **Read the instructions.** For example, some integration problems will just tell you to set up an integral and go on. **Don’t waste time integrating those!**

(d) Be very solid on the coordinate transformations.

(e) When in doubt, **think about what things mean!**