1. Let \( y = x^r \). We get \( y' = rx^{r-1} \) and \( y'' = r(r-1)x^{r-2} \), and \( \alpha = 4, \beta = 2 \). Thus, the differential equation becomes \( x'(r^2 + (4-1)r + 2) = 0 \). Therefore, \( r^2 + 3r + 2 = 0 \), so \( r = -1 \) or \( r = -2 \). The general solution is \( y = c_1x^{-1} + c_2x^{-2} \). We do not need \( |x| \) since \( c_1 \) can absorb the sign of \( x \).

2. Let \( y = x^r \). We have \( \alpha = -3 \) and \( \beta = 4 \), so proceeding as usual gives us \( x'(r^2 + (-3-1)r + 4) = 0 \), and we get the repeated root \( r = 2 \). The general solution is thus \( y(x) = c_1x^2 + c_2x^2\ln|x| \). (The \( x^2 \) obviates the need for an absolute value.)

3. We have \( \alpha = -1 \) and \( \beta = 1 \). Thus \( r^2 + (-1-1)r + 1 = 0 \), so \( r = 1, 1 \). The general solution is \( c_1x + c_2x \ln|x| \). The constants can absorb the sign of \( x \).

4. We have \( \alpha = 6 \) and \( \beta = -1 \). Thus \( r^2 + (6-1)r - 1 = 0 \), so \( r = -5, \frac{1}{2} \). The general solution is \( c_1|x|^{(-5+\sqrt{29})/2} + c_2|x|^{(-5-\sqrt{29})/2} \).

5. We have \( \alpha = -5 \) and \( \beta = 9 \). Thus \( r^2 + (-5-1)r + 9 = 0 \), so \( r = 3, 3 \). The general solution is \( c_1x^3 + c_2x^3\ln|x| \).

6. We have \( \alpha = 2 \) and \( \beta = 4 \). Thus \( r^2 + (2-1)r + 4 = 0 \), so \( r = -1, 1 \). The general solution is \( c_1|x|^{-1/2} \cos \left( \frac{\sqrt{15}}{2} \ln|x| \right) + c_2|x|^{-1/2} \sin \left( \frac{\sqrt{15}}{2} \ln|x| \right) \).

7. We have \( \alpha = 1 \) and \( \beta = -\frac{3}{2} \). Thus \( r^2 + (1 - 1) r - \frac{3}{2} = 0 \), or \( 2r^2 - r - 3 = 0 \), so \( r = 3/2 \) or \( r = -1 \). The general solution is \( y(x) = c_1|x|^{3/2} + c_2x^{-1} \). We require \( y(1) = 1 \), so \( c_1 + c_2 = 1 \). Also, since \( 1 > 0 \), we may use \( y(x) = c_1x^{3/2} + c_2x^{-1} \). This gives \( y'(x) = \frac{3}{2}c_1x^{1/2} - c_2x^{-2} \), so \( y'(1) = \frac{3}{2}c_1 - c_2 = 4 \).

8. We have \( \alpha = 0 \) and \( \beta = 2 \). Thus \( r^2 + (0-1)r + 4 = 0 \), so \( r = 1, -4 \). If \( 1 - 4\beta = 0 \), then the solutions are \( |x|^{1/2} \) and \( |x|^{-1/2} \ln|x| \); these both approach zero as \( x \to 0 \). Now suppose \( 1 - 4\beta > 0 \). We need both solutions to be positive (so that \( x \) has a positive exponent). The negative root gives the smaller smaller solution, so it is sufficient to guarantee that this one is positive.

\[
1 - \sqrt{1 - 4\beta} > 0
\]
\[
1 - 1 - 4\beta > 0
\]
\[
1 > \sqrt{1 - 4\beta}
\]
\[
1 > 1 - 4\beta
\]
\[
0 > -4\beta
\]
\[
4\beta > 0.
\]
Finally, if $1 - 4\beta < 0$, then we get $|x|^{1/2} \cos \left( \frac{\sqrt{4\beta - 1}}{2} \ln |x| \right)$ and a similar solution with a sine function. Both approach zero as $x \to 0$.

Thus, the only condition is that $\beta > 0$.

19. We may use our solution from Number 18 with $\beta = -2$: $y(x) = c_1 x^2 + c_2 x^{-1}$. Since $y(1) = 1$, $c_1 + c_2 = 1$.
   Since $y'(1) = \gamma$, $2c_1 - c_2 = \gamma$. Solving gives $c_1 = \frac{\gamma + 1}{3}$ and $c_2 = \frac{2 - \gamma}{3}$. In order to have a finite limit, we need to get rid of the $x^{-1}$ term, so we choose $\gamma = 2$. 