1. (a) \[
\begin{array}{cc|cc}
+ & 0 & 1 \\
0 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 \\
\end{array}
\]

(b) See back of book.

2. (a) In \( \mathbb{Z}_8 \), \( 0^2, 2^2, 4^2, 6^2 \neq 1 \), but \( 1^2 = 3^2 = 5^2 = 7^2 = 1 \).

(b) In \( \mathbb{Z}_6 \), \( 0^4 \neq 1 \), but \( 1^4 = 2^4 = 3^4 = 4^4 = 1 \).

(c) In \( \mathbb{Z}_6 \), \( 0^2 + 3 \cdot 0 + 2 = 2, 1^2 + 3 \cdot 1 + 2 = 0, 2^2 + 3 \cdot 2 + 2 = 0, 3^2 + 3 \cdot 3 + 2 = 2, 4^2 + 3 \cdot 4 + 2 = 0, \) and \( 5^2 + 3 \cdot 5 + 2 = 0 \). Thus 1, 2, 4, and 5 are all solutions.

(d) Since we are looking for \( x \) such that \( 12|x^2 + 1 \), \( x \) will have to be odd. Also, since \( x^2 = (-x)^2 \), we need only consider 1, 3, and 5: \( 7 = -5, 9 = -3, \) and \( 11 = -1 \). Now \( 1^2 + 1 \neq 0, 3^2 + 1 \neq 0, \) and \( 5^2 + 1 \neq 0 \), so there are no solutions.

3. (a) Certainly 0 and 1 can’t work. Consider 2: \( 2^1 = 2, 2^2 = 4, 2^3 = 1, 2^4 = 2, \ldots \) and now we’re just going to cycle. Try 3: \( 3^1 = 3, 3^2 = 2, 3^3 = 6, 3^4 = 4, 3^5 = 5, 3^6 = 1 \). That works!

(b) Try 2: \( 2^1 = 2, 2^2 = 4, 2^3 = 3, 2^4 = 1 \). That works, too!

(c) Try 2: \( 2^1 = 2, 2^2 = 4, 2^3 = 2, \ldots \) Try 3: \( 3^1 = 3, 3^2 = 3, \ldots \) Try 4: \( 4^1 = 4, 4^2 = 4, \ldots \) Try 5: \( 5^1 = 5, 5^2 = 1, 5^3 = 5, \ldots \) Nothing works!

10. 0 is not such a number for any \( \mathbb{Z}_n \) since \( 0x = 0 \).

(a) \( 1 \cdot 1 = 1, 2 \cdot 3 = 1, 4 \cdot 4 = 1 \), so everything except 0 will work.

(b) \( 1 \cdot 1 = 1 \) and \( 3 \cdot 3 = 1 \), but \( 2 \cdot 0 = 0, 2 \cdot 1 = 2, 2 \cdot 2 = 0, \) and \( 2 \cdot 3 = 2 \). Thus, only 1 and 3 work.

(c) \( 1 \cdot 1 = 1, 2 \cdot 2 = 1 \).

(d) \( 1 \cdot 1 = 1 \) and \( 5 \cdot 5 = 1 \). However, from the multiplication tables on page 33, we see that these are the only two.