1. (a) \( f(x) - g(x) = x^5 - 5x^4 + 2x^3 + 5x^2 - 3x - 1 = (x^2 + 1)(x^3 - 5x^2 + x + 10) - 4x - 11 \). \( f(x) \) and \( g(x) \) are not congruent mod \( p(x) \). (We could also have checked to see whether \( i \) is a root. It isn’t.)

(b) \( f(x) - g(x) = x^3 + 2x^2 - 11x + 20 = (x^2 - 3x + 4)(x + 5) \), so \( f(x) \equiv g(x) \pmod{p(x)} \).

(c) \( f(x) - g(x) = x^5 - 2x^4 + 4x^3 - 8x^2 + 3x - 2 \) looks like a nuisance to divide, but notice that 1 is a root of \( p(x) \) and not a root of \( f(x) - g(x) \). Therefore, \( f(x) \) is not congruent to \( g(x) \) mod \( p(x) \).

3. According to Corollary 5.5, the set of polynomials of degree at most 2 in \( \mathbb{Z}_2[x] \) gives a complete set of representatives of the congruence classes mod \( x^3 + 2x + 1 \). These have the form \( ax^2 + bx + c \). Since there are two choices for each coefficient and there are three coefficients, there are \( 2^3 = 8 \) such polynomials.

6. Since the only polynomials of degree less than 1 are constant polynomials, constants give a complete set of congruence classes modulo \( x - a \).

10. This is true: Suppose \( p(x) \) is irreducible in \( F[x] \) and \( f(x)g(x) \equiv 0_F \pmod{p(x)} \). Then \( p(x) | f(x)g(x) \), and, since \( p(x) \) is irreducible, we have that \( p(x) | f(x) \) or \( p(x) | g(x) \) by Theorem 4.11. Thus \( f(x) \equiv 0_F \pmod{p(x)} \) or \( g(x) \equiv 0_F \pmod{p(x)} \).

13. If \( x | f(x) - g(x) \), then \( f(x) \) and \( g(x) \) differ by only a constant. Therefore, the graph of \( y = g(x) \) is just a vertical translation of the graph of \( y = f(x) \).