Group Exam 1

Name: KEY

Math 142: Calculus II

Name of group member:

Spring 2008, Professor McNicholas

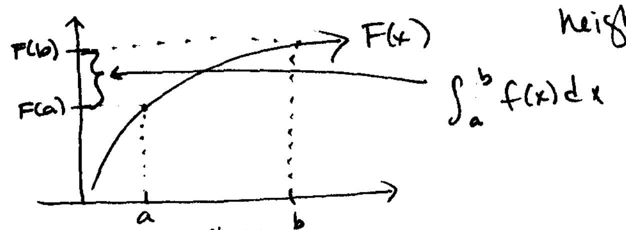
Name of group member:

Show your work and make sure your answers are well organized, easy to follow, and properly explained.

Problem 1:

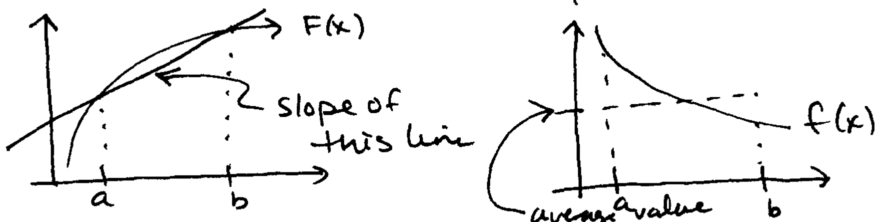
Given $F'(x) = f(x)$, use the Fundamental Theorem of Calculus to answer each of the following. For each question, complete the statement and give an illustration on the provided graph.

(a) On the graph of $F(x)$, $\int_a^b f(x)dx$ represents $F(b) - F(a)$, i.e. the change in height from a to b.



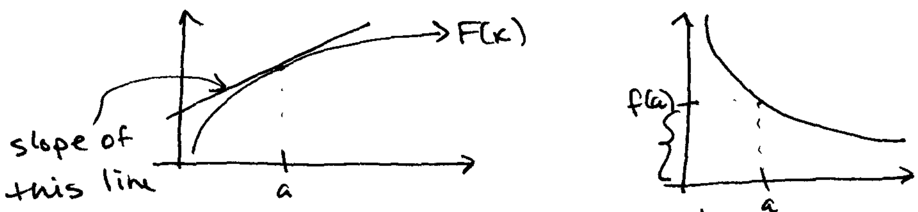
(b) On the graph of $f(x)$, $\frac{\int_a^b f(x)dx}{b-a}$ represents the average value from a to b.

On the graph of $F(x)$, $\frac{\int_a^b f(x)dx}{b-a}$ represents the slope of the secant line passing through pnts $(a, F(a))$ & $(b, F(b))$

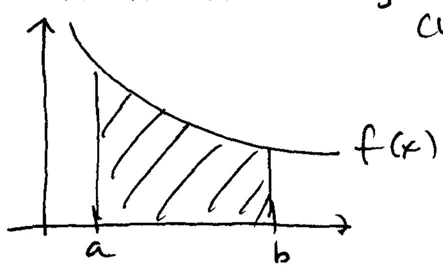


(c) On the graph of $F(x)$, $F'(a)$ represents Slope of the tangent line @ $x=a$

On the graph of $f(x)$, $F'(a)$ represents $f(a)$



(d) On the graph of $f(x)$, $F(b) - F(a)$ represents $\int_a^b f(x)dx$, i.e. the area under the curve from a to b.



Signature line:

$\boxed{\text{Group Exam 1}}$ Name: KEY

Math 142: Calculus II — Name of group member: ______

Spring 2008, Professor McNicholas — Name of group member: ______

Show your work and make sure your answers are well organized, easy to follow, and properly explained.

Problem 2:

Solve each of the following indefinite integrals

(a) $\int \frac{\ln(e^{2x})}{x^2}dx$ $= \int \frac{2x}{x^2}dx$ $\quad u = x^2$, $du = 2x\,dx$ $\quad \int \frac{du}{u} = \ln|u| + C$

$$\boxed{= \ln|x^2| + C}$$

(b) $\int \frac{x}{\sqrt{1-x^2}}dx$ $\quad u = 1-x^2$, $du = -2x\,dx$, $-\frac{1}{2}du = x\,dx$ $\longrightarrow \int \frac{-\frac{1}{2}du}{u^{1/2}} = -\frac{1}{2}\left(2u^{1/2}\right) + C$

$$\boxed{= -\sqrt{1-x^2} + C}$$

(c) $\int \frac{e^x}{1+e^{2x}}dx$ $\quad u = e^x$, $du = e^x dx$ $\rightarrow$ $\int \frac{du}{1+u^2} = \arctan(u) + C$

$$\boxed{= \arctan(e^x) + C}$$

(d) $\int x^5\sqrt{x^2-2}dx$ $\quad u = x^2 - 2 \rightarrow u + 2 = x^2$

$\quad ||$ $\qquad du = 2x\,dx$

$\int x^4\sqrt{x^2-2}\,(x\,dx)$ $\quad \frac{1}{2}du = x\,dx$

$\longrightarrow \frac{1}{2}\int (u+2)^2\sqrt{u}\,du = \frac{1}{2}\int (u^2+4u+4)u^{1/2}du$

$= \frac{1}{2}\int (u^{5/2} + 4u^{3/2} + 4u^{1/2})du = \frac{1}{2}\left[\frac{2}{7}u^{7/2} + 4\left(\frac{2}{5}\right)u^{5/2} + 4\left(\frac{2}{3}\right)u^{3/2}\right] + C$

$$\boxed{= \frac{1}{7}(x^2-2)^{7/2} + \frac{4}{5}(x^2-2)^{5/2} + \frac{4}{3}(x^2-2)^{3/2} + C}$$

Signature line: ______________________ ______________________

$\boxed{\text{Group Exam 1}}$	Name: KEY
Math 142: Calculus II	Name of group member:
Spring 2008, Professor McNicholas	Name of group member:

Show your work and make sure your answers are well organized, easy to follow, and properly explained.

Problem 3:

(a) Find the following definite integral

$$\int_0^{\pi} \cos^3 x \sin x dx$$

$u = \cos(x)$
$du = -\sin(x)dx$
$x = 0 \rightarrow u = 1$
$x = \pi \rightarrow u = -1$

$$\downarrow$$

$$\int_1^{-1} u^3(-du) = -\frac{1}{4}u^4\Big|_1^{-1} = -\frac{1}{4}\left((-1)^4 - (1)^4\right) = 0$$

(b) Find the equation of the line tangent to the graph of $F(x) = \int_0^x t^2(t^3+1)^4 dt$ at $x = 1$.

Slope $= F'(1) = 1^2(1^3+1)^4 = 16$

point on the line : $(1, F(1))$

$$F(1) = \int_0^1 t^2(t^3+1)^4 dt$$

$u = t^3+1 \rightarrow t = 0 \rightarrow u = 1$
$du = 3t^2dt \quad t = 1 \rightarrow u = 2$
$\frac{1}{3}du = t^2dt$

$$= \int_1^2 u^4\left(\frac{1}{3}du\right)$$

$$= \frac{1}{3}\int_1^2 u^4du = \frac{1}{3}\left[\frac{1}{5}u^5\right]_1^2 = \frac{1}{15}(2^5 - 1^5) = \frac{1}{15}(32-1) = \frac{31}{15}$$

EQN of Line: $\boxed{\left(y - \frac{31}{15}\right) = 16(x-1)}$ ← point-slope form

(c) If you used your answer to part (b) to approximate the value of $F(1.1)$ would your approximation be too large or too small? Use mathematical reasoning to justify your answer.

$$F''(t) = 2t(t^3+1)^4 + 4t^2(t^3+1)^3 3t^2$$

$F''(1) = 2(16) + 4(8)(12)$ ← positive

so $F(x)$ is concave up @ $x = 1$ ⇒ answer to would be too small.

Signature line: ______________________ ______________________