

**Group Exam 3**

Name: KEY

Math 142: Calculus II

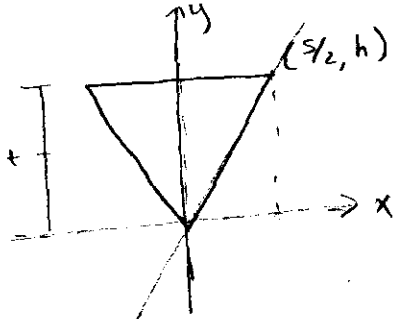
Name of group member: \_\_\_\_\_

Spring 2008, Professor McNicholas

Name of group member: \_\_\_\_\_

Show your work and make sure your answers are well organized, easy to follow, and properly explained.  
**Problem 1:**

Imagine an inverted square pyramid with base side  $s$  and height  $h$ . At what vertical height would you have to cut off the pyramid's point to get two pieces of equal volume? (Hint: Problem is much easier if you measure height from the pyramid's tip, not base)



Let  $t$  be the height @ which you divide the pyramid into two parts of equal volume,  
 i.e.

{ 2pts

$$y = \frac{h}{s/2} x = \frac{2h}{s} x$$

$$\Rightarrow x = \frac{s}{2h} y$$

width @ height  $y$   
 $= 2x = \frac{s}{h} y$

$$\int_0^t \left(\frac{s}{h} y\right)^2 dy = \int_t^h \left(\frac{s}{h} y\right)^2 dy \quad \left. \vphantom{\int_0^t} \right\} 15pts$$

$$\frac{s^2}{3h^2} y^3 \Big|_0^t = \frac{s^2}{3h^2} y^3 \Big|_t^h$$

$$\frac{s^2}{3h^2} t^3 = \frac{s^2}{3h^2} [h^3 - t^3]$$

$$t^3 = h^3 - t^3$$

$$2t^3 = h^3$$

$$t = \frac{h}{\sqrt[3]{2}}$$

{ 15pts

Signature line: \_\_\_\_\_

**Group Exam 3**

Name: KEY

Math 142: Calculus II

Name of group member: \_\_\_\_\_

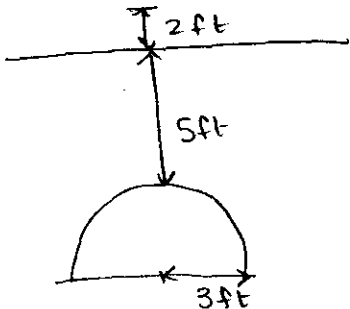
Spring 2008, Professor McNicholas

Name of group member: \_\_\_\_\_

Show your work and make sure your answers are well organized, easy to follow, and properly explained.

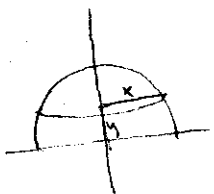
**Problem 2:**

A hemispherical gasoline tank with radius 3ft is buried under a service station with its flat side down. The top of the tank is 5 ft underground. Find the work needed to pump all the gasoline in the tank out a nozzle that is 2ft above ground. (Note: Gasoline weighs 42 lb/ft<sup>3</sup>.)



$$\Delta W = (\Delta \text{Volume}) (\text{weight/volume}) (\text{Distance}) \leftarrow 10 \text{pts}$$

$$= (\pi(9-y^2)\Delta y) (42 \text{ lb/ft}^3) (2+5+3-y) \leftarrow 20 \text{pts}$$



$$x^2 + y^2 = 9$$

$$x = \sqrt{9-y^2}$$

Area of cross section

$$\pi r^2 = \pi (\sqrt{9-y^2})^2$$

$$= \pi(9-y^2)$$

10pts

$$W = \int_0^3 42\pi(9-y^2)(10-y)dy$$

$$= 42\pi \int_0^3 (y^3 - 10y^2 - 9y + 90)dy$$

$$= 42\pi \left[ \frac{1}{4}y^4 - \frac{10}{3}y^3 - \frac{9}{2}y^2 + 90y \right]_0^3$$

$$= 42\pi \left[ \frac{1}{4}(3^4) - \frac{10}{3}(3^3) - \frac{9}{2}(3^2) + 90 \cdot 3 \right]$$

$$= 6709.5\pi$$

Signature line: \_\_\_\_\_

**Group Exam 3**

Name: KEY

Math 142: Calculus II

Name of group member: \_\_\_\_\_

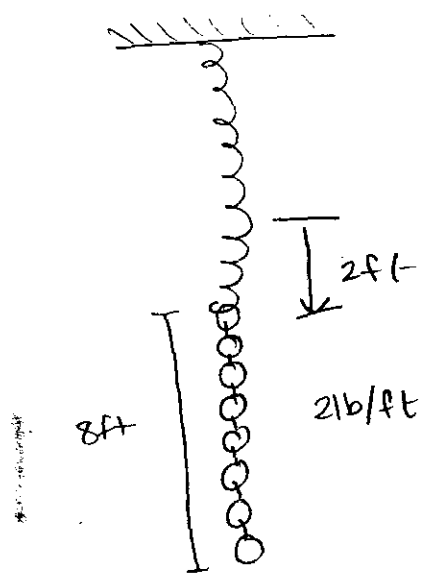
Spring 2008, Professor McNicholas

Name of group member: \_\_\_\_\_

Show your work and make sure your answers are well organized, easy to follow, and properly explained.

**Problem 3:**

One end of a spring is attached to the ceiling and a chain is hung from the other end. The chain is 8 feet long and weighs 2 lb/ft. The end not attached to the spring just brushes the floor. The weight of the chain causes the spring to stretch 2 ft from its equilibrium position. Find the work done in pulling down on the chain a distance of 4 feet.



10pts  
 Force = Force down on spring - help from chain

Spring Force:  $F_s(x) = kx$   
 $F_s(2) = k(2) = (8ft)(2lb/ft)$   
 $k = 8$  } 10pts

Force =  $8x - (8 - (x - 2))2$   
 $= 8x - (10 - x)2 = 8x - 20 + 2x$   
 $= 10x - 20$  ← 10pts

10pts  
 $w = \int_2^6 (10x - 20) dx$   
 $= 5x^2 - 20x \Big|_2^6 = 60 - (-20) = 80$

Alt:  
 $8(x+2) - (8-x)2$   
 $= 8x + 16 - 16 + 2x = 10x$

$\int_0^4 10x dx = 5x^2 \Big|_0^4 = 80$

Signature line: \_\_\_\_\_