Math 251: Foundations of Advanced Mathematics

Solutions to Chapter 1.3

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Above I have have demonstrated a possible heading for your homework solutions. You should feel free to format your header however you like. You might want to use textboxes, *italic script*, **bold face**, underline, etc. In this document, I will try to demonstrate some of the more common commands you'll be using in LaTeX.

To begin with, lets cover some different text sizes:

This is Huge

This is LARGE

This is Large

This is normalsize

This is small

This is tiny

You have to use two returns in LaTeXto start a new line. All new lines will be indented unless you start with a "noindent" command.
The following list was created using itemize.

- Variables and symbols should all be in math mode, but regular text should not. LaTeXwill choose font types and spacing based on which mode you are in, so you want to make sure text is in text mode and math is in math mode.
- Now lets look at some examples using tables. The "begin{array}" command sets the number of columns. In the commands for tables and arrays, the c, l, and r refer to the alignment of the entries in each column: center, left, or right. Columns are separated by ampersands (&) and rows are ended with double backslashes (\\).

$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$	$\mathbb{C}$
Natural Numbers	Integers	Rationals	Reals	Complex

P	A	B	P	$\rightarrow$	$(A$	$\vee$	$B)$	$\leftrightarrow$	$(P$	$\wedge$	$\neg A)$	$\rightarrow$	B
T	T	T	T	T	T	T	T	T	T	F	F	T	T
T	T	F	T	T	T	T	F	T	T	F	F	T	F
T	F	T	T	T	F	T	T	T	T	T	T	T	T
T	F	F	T	F	F	F	F	T	T	T	T	F	F
F	T	T	F	T	T	T	T	T	F	F	F	T	T
F	T	F	F	T	T	T	F	T	F	F	F	T	F
F	F	T	F	T	F	T	T	T	F	F	T	T	T
F	F	F	F	T	F	F	F	T	F	F	T	T	F
			1	8	2	7	3	11	4	9	5	10	6

$P \rightarrow Q$
$R \wedge \neg Q$

$\neg Q$ $\quad$ $Simplification$
$\neg P$ $\quad$ $Contrapositive$

The "mbox" command produces normal text in the math setting, while $ signs produce math mode text. You can also switch into math mode using the "begin{equation}" or "begin{eqnarray}" commands.

$$e^7 \geq x_{i^2-2} \tag{1}$$

$$\begin{aligned} \sin(\frac{x\pi}{6}) &= \ln(y^2) && (2)\\ 2y &= 5f(x) && (3)\\ 12z &\leq \sum_{n=0}^{\infty} f(x_n) \\ &\leq g(z) && (4) \end{aligned}$$

"\begin{eqnarray}" is primarily used to generate a list of aligned equations. To get the previous equation array I entered

```
\begin{eqnarray}
\sin(\frac{x\pi}{6})&=&\ln(y^2)\\
2y&=&5f(x)\\\nonumber
12z &\le&\sum_{n=0}^{\infty}f(x_n)\\
&\le& g(z)
\end{eqnarray}
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. The & symbol is used to distinguish where the equations are to be aligned, the \\ symbol is used to end a line in the equation array.

Here are some other examples:

Let K be the statement "you will kiss me," and let J be the statement "I will dance a jig." The question is whether $(K \to J) \wedge J$ implies K, which it does not. You can verify it doesn't using a truth table. The failing case is when K is false, and J is true. Thus $(K \to J) \wedge J$ is true while K is false.

1. $e^5 \leq 0$
2. $3 \geq 5$ and $7 < 8$
3. $\sin(\pi/2) \geq 0$ or $\tan(0) < 0$
4. $y = 3$ and $y^2 \neq 7$
5. $w - 3 > 0$ and $w^2 + 9 \leq 6w$
6. $a - b = c$ and $a \neq b + c$, or $a - b \neq c$ and $a = b + c$

$\neg(P \to \neg Q) \Leftrightarrow P \wedge Q$

$A \to (A \wedge B) \Leftrightarrow A \to B$

Tautology $(X \wedge Y) \to X \Leftrightarrow X \to X$

Contradiction $\neg(M \vee L) \wedge L) \Leftrightarrow \neg L \wedge L$

$(P \to Q) \vee Q \Leftrightarrow P \to Q$

$\neg(X \to Y) \vee Y \Leftrightarrow (X \wedge \neg Y) \vee Y \Leftrightarrow X \vee Y$

Here's a sample proof.

proposition: If P is an invertible nxn matrix and C is any kxn matrix, then $rank(CP) = rank(C)$ and $nullity(CP) = nullity(C)$.

proof For all matrices $rank(A) = rank(A^T)$. Thus, $rank(CP) = rank((CP)^T) = rank(P^T C^T)$. If P is invertible, then so too must P^T be. Thus, by our B.K-A.U. Theorem, P^T is expressible as the product of elementary matrices. Multiplying C^T by P^T is equivalent to performing elementary row operations on C^T, which by theorem 7.3.7 does not change the row space or null space. Thus, $rank(CP) = rank((CP)^T) = rank(P^T C^T) = dim(row(P^T C^T)) = dim(row(C^T)) = rank(C^T) = rank(C)$. By the dimension theorem and our previous result, $nullity(CP) = k - rank(CP) = k - rank(C) = nullity(C)$. ■