Group Exam 2	Name: KEY
Math 141-2, Calculus I	Name of group member:
Professor McNicholas	Name of group member:

Show your work and make sure your answers are well organized, easy to follow, and properly explained.

Problem 1:
On the following graph of $f(x)$, clearly mark the lengths and slopes which represent each of the following:

(a) h

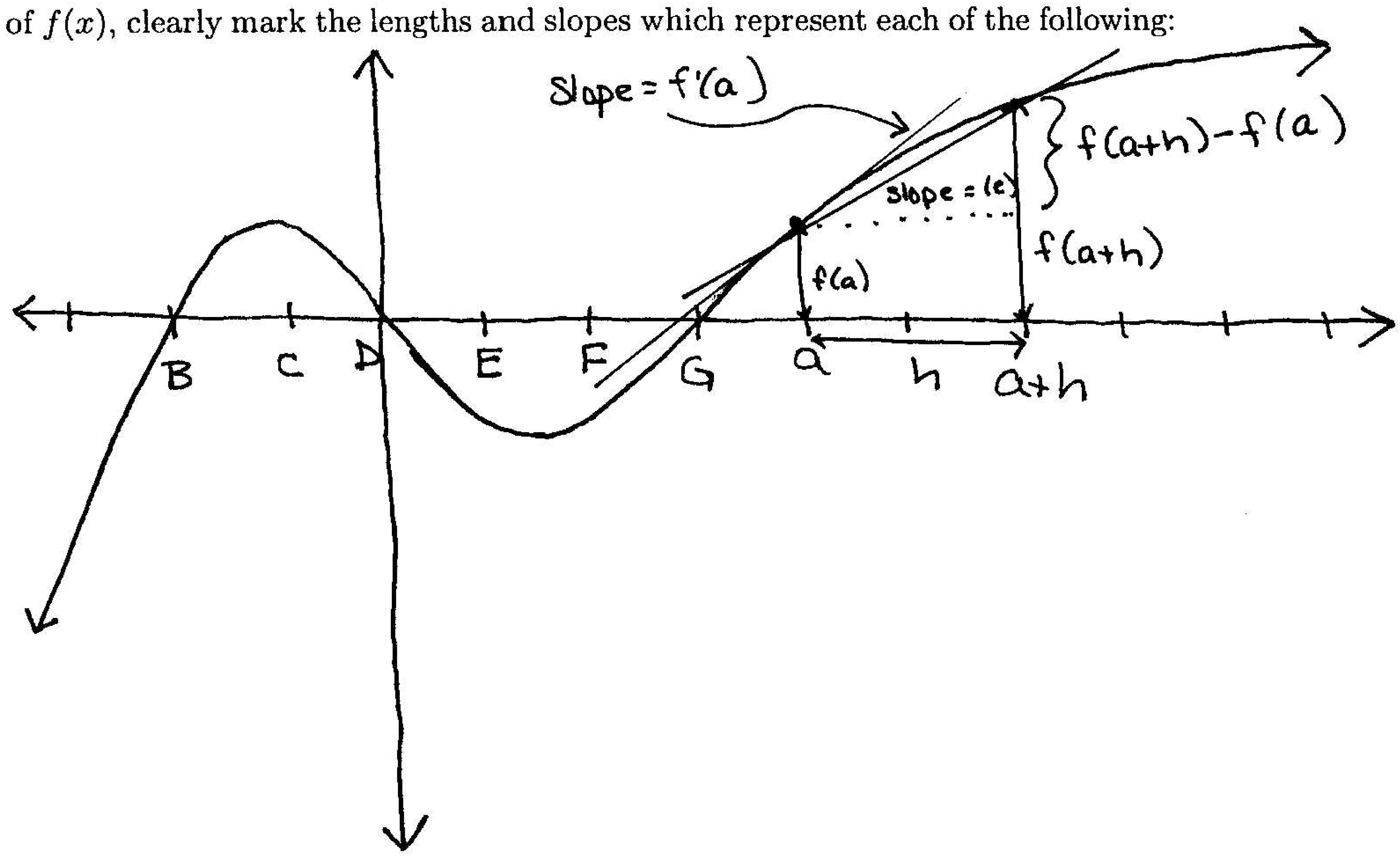
(b) $f(a)$

(c) $f(a+h)$

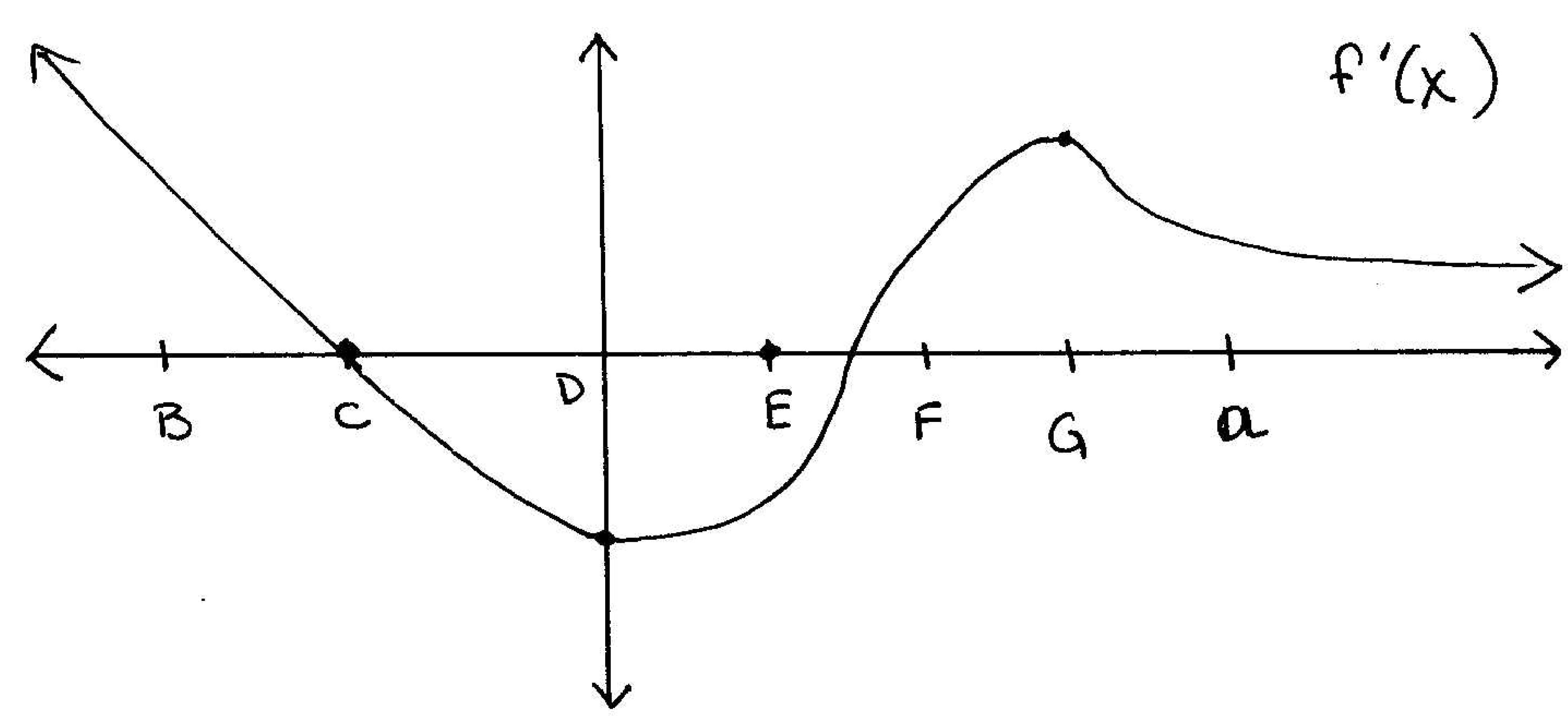
(d) $f(a+h) - f(a)$

(e) $\frac{f(a+h)-f(a)}{h}$

(f) $f'(a)$



Draw a graph of the derivative function $f'(x)$. Label the x-intercepts



Signature line: ______________________ ______________________

Group Exam 2	Name: Key
Math 141-2, Calculus I	Name of group member:
Professor McNicholas	Name of group member:

Show your work and make sure your answers are well organized, easy to follow, and properly explained.

Problem 2:
The distance traveled in t seconds by a particle is described by the function $d(t) = 2t^3$, where $d(t)$ gives the distance traveled in centimeters.

(a) Use the limit definition of the derivative to find the function $v(t)$ describing the velocity of the particle at time t.

$$v(t) = d'(t) = \lim_{\varepsilon \to 0} \frac{d(t+\varepsilon) - d(t)}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{2(t+\varepsilon)^3 - 2t^3}{\varepsilon}$$

$$= \lim_{\varepsilon \to 0} \frac{2(t^3 + 3t^2\varepsilon + 3t\varepsilon^2 + \varepsilon^3) - 2t^3}{\varepsilon}$$

$$= \lim_{\varepsilon \to 0} \frac{\cancel{2t^3} + 6t^2\varepsilon + 6t\varepsilon^2 + 2\varepsilon^3 - \cancel{2t^3}}{\varepsilon}$$

$$= \lim_{\varepsilon \to 0} \frac{\cancel{\varepsilon}(6t^2 + 6t\varepsilon + 2\varepsilon^2)}{\cancel{\varepsilon}} = \lim_{\varepsilon \to 0} 6t^2 + 6t\varepsilon + 2\varepsilon^2 \boxed{= 6t^2}$$

(b) Given $d(2) = 16$ and $d'(2) = 24$, what are the units of 2, 16, and 24?

2 seconds

16 centimeters

24 cm/s

(c) Interpret the meaning of $d'(2) = 24$ in terms of the distance traveled.

After 2 seconds, the particle is moving 24 cm/s.

– or equivalently –

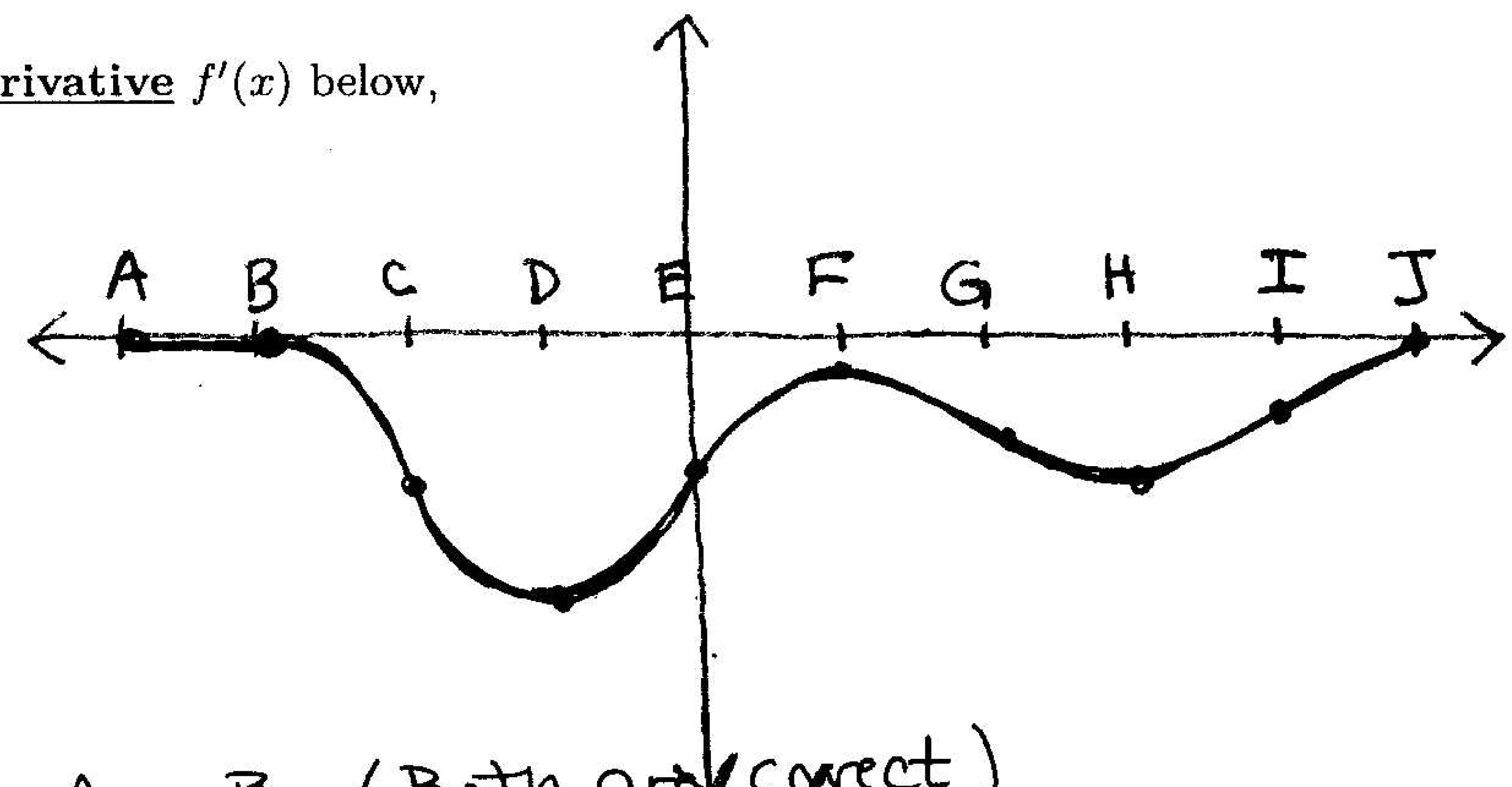
After 2 seconds, in an additional second the particle will move an additional 24 cm.

Signature line: ______________________ ______________________

Group Exam 2	Name: Key
Math 141-2, Calculus I	Name of group member:
Professor McNicholas	Name of group member:

Show your work and make sure your answers are well organized, easy to follow, and properly explained.

Problem 3:
Given the graph of the **derivative** $f'(x)$ below,



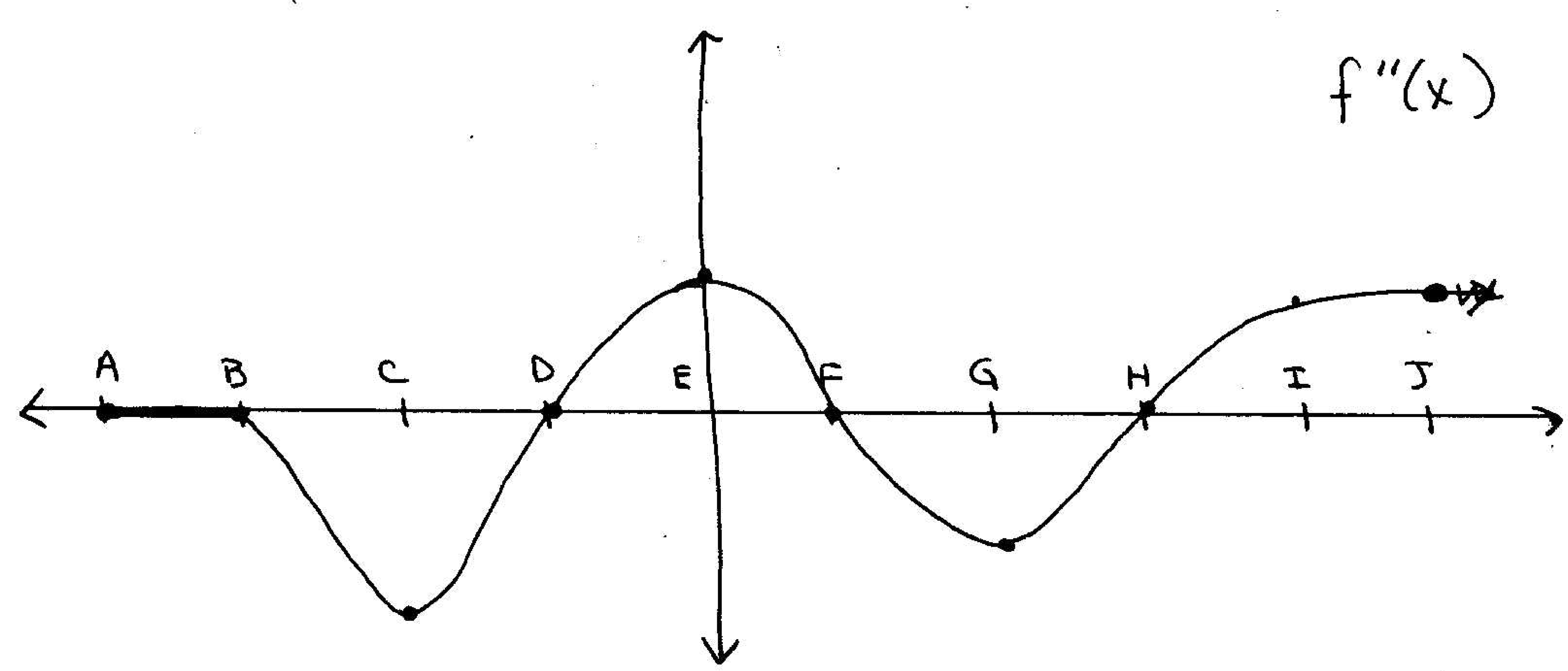
(a) Where is $f(x)$ greatest? A or B (Both are correct)

(b) Where is $f(x)$ smallest? J

(c) Where is $f''(x)$ smallest (i.e. most negative)? C

(d) Where is $f''(x)$ largest? E

(f) Draw the graph of $f''(x)$.



(g) Fill in the blanks. If $f''(x) > 0$ over some interval, $f'(x)$ is increasing over that interval, and $f(x)$ is concave up over that interval.

Signature line: ______________ ______________

Group Exam 2	Name: KEY
Math 141-3, Calculus I	Name of group member:
Professor McNicholas	Name of group member:

Show your work and make sure your answers are well organized, easy to follow, and properly explained.

Problem 1:
On the following graph of $f(x)$, clearly mark the lengths and slopes which represent each of the following:

(a) h

(b) $f(a)$

(c) $f(a+h)$

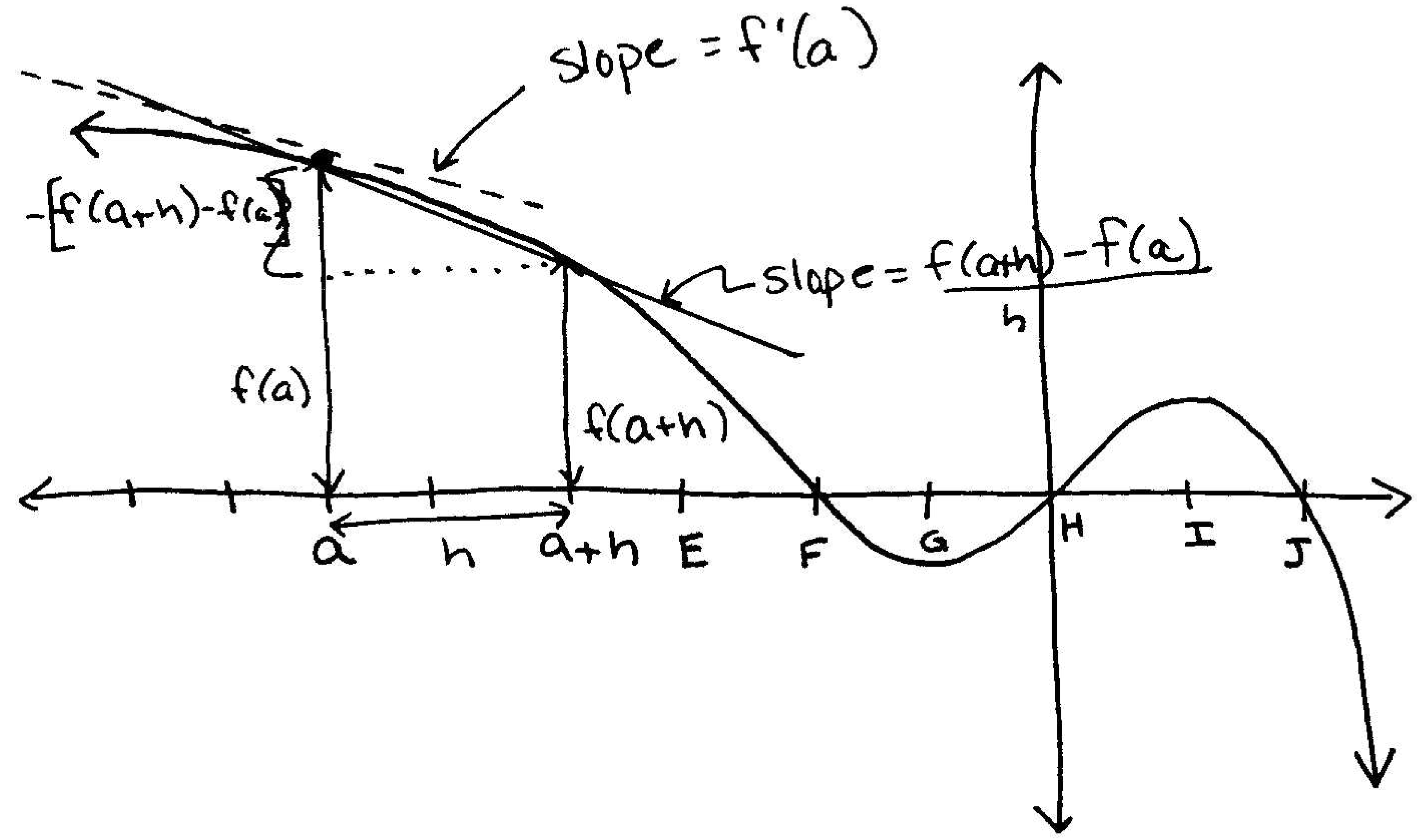
(d) $f(a+h) - f(a)$

(e) $\frac{f(a+h)-f(a)}{h}$

(f) $f'(a)$

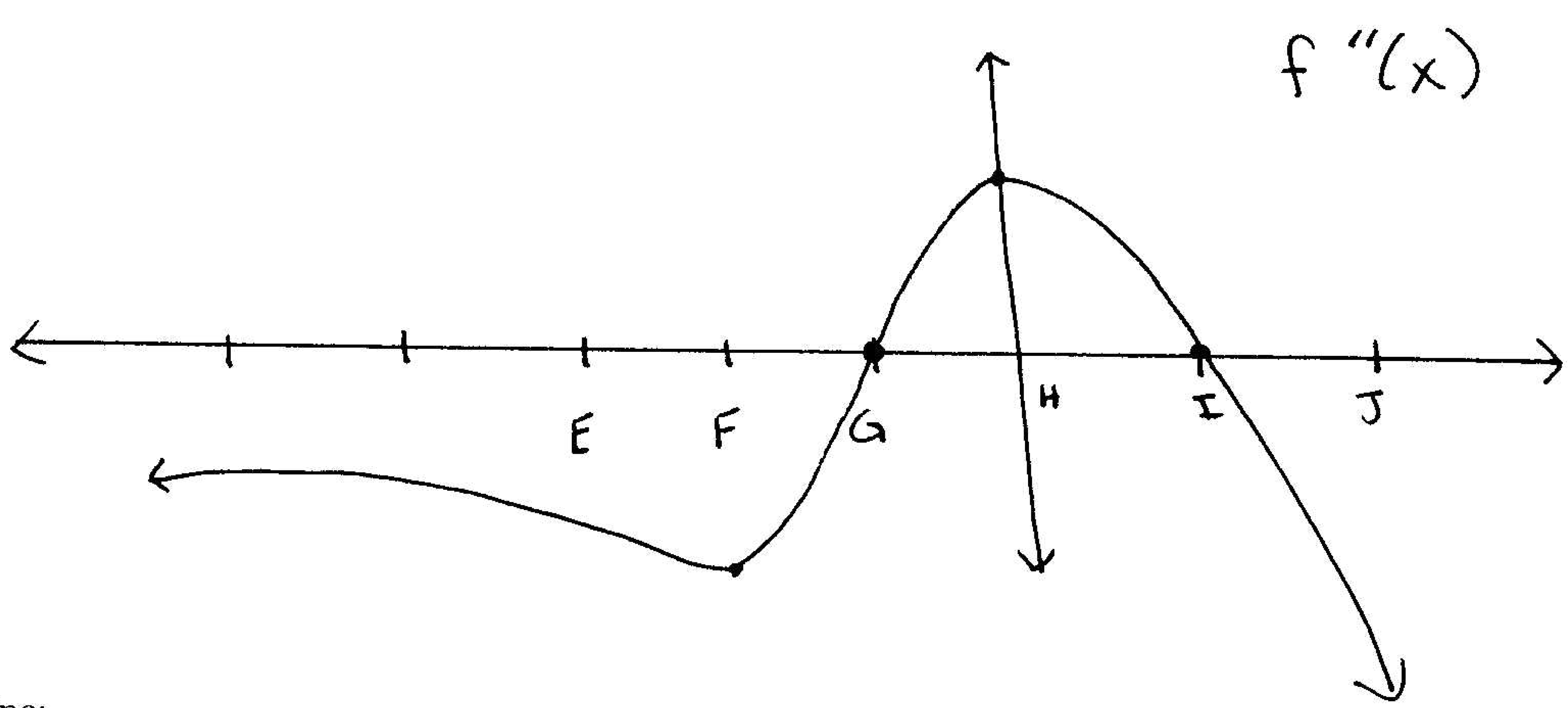
3pts ea

2pts each



Draw a graph of the derivative function $f'(x)$. Label the x-intercepts

6pts



Signature line: ______________________ ______________________

Group Exam 2

Name: KEY

Math 141-3, Calculus I

Name of group member:

Professor McNicholas

Name of group member:

Show your work and make sure your answers are well organized, easy to follow, and properly explained.

Problem 2:

(a) Use the limit definition and successively smaller values of ϵ to find $f'(0)$ given $f(x) = 5^x$.

$$f'(0) = \lim_{\varepsilon \to 0} \frac{f(0+\varepsilon) - f(0)}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{5^{\varepsilon} - 5^{0}}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{5^{\varepsilon} - 1}{\varepsilon}$$

$$\approx \frac{5^{0.00001} - 1}{0.00001} = 1.6094$$

(b) Use the limit definition of the derivative, algebraic manipulation, and your answer to part (a) to show that $f'(x) = 5^x \ln(5)$.

$$f'(x) = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon) - f(x)}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{5^{x+\varepsilon} - 5^x}{\varepsilon}$$

$$= \lim_{\varepsilon \to 0} \frac{5^x \cdot 5^{\varepsilon} - 5^x}{\varepsilon} = \lim_{\varepsilon \to 0} 5^x \left(\frac{5^{\varepsilon} - 1}{\varepsilon}\right)$$

$$= 5^x \lim_{\varepsilon \to 0} \frac{5^{\varepsilon} - 1}{\varepsilon} \approx 5^x (1.6094)$$

note: $\ln(5) = 1.609437912$

$$\Rightarrow f'(x) = 5^x \ln(5).$$

Signature line: ______________________ ______________________

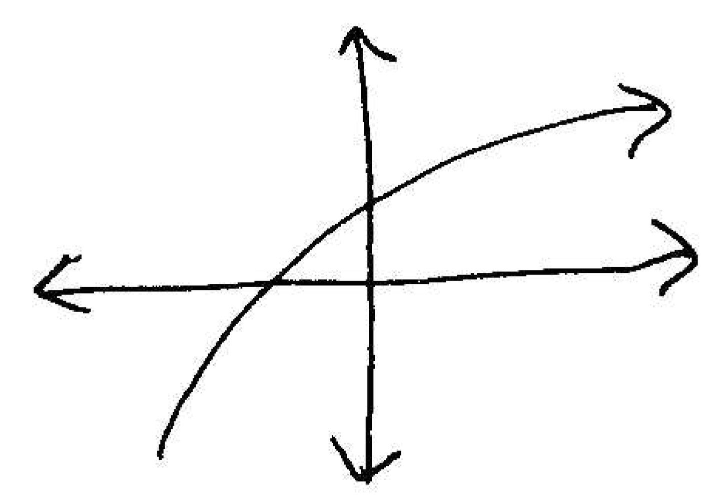
Group Exam 2	Name: KEY
Math 141-3, Calculus I	Name of group member:
Professor McNicholas	Name of group member:

Show your work and make sure your answers are well organized, easy to follow, and properly explained.

Problem 3:
Assume f and g are differentiable functions defined on all real numbers. Determine whether the following are true or false. If true, sketch a drawing showing a scenario in which the statement is true. If false, sketch a drawing showing a scenario in which the statement is false.

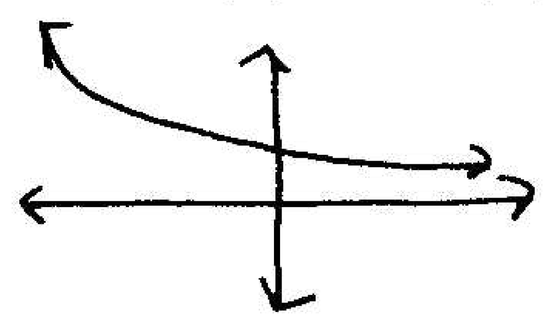
(a) It is possible that for all x, $f(x) > 0$, $f'(x) > 0$, and $f''(x) < 0$.

False. must have this shape



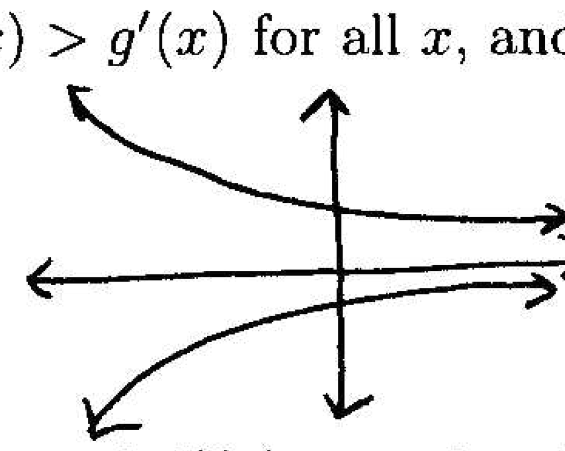
(b) It is possible that for all x, $f(x) > 0$, $f'(x) < 0$, and $f''(x) > 0$.

True



(c) It is possible for $f'(x) > g'(x)$ for all x, and $f(x) < g(x)$ for all x.

True

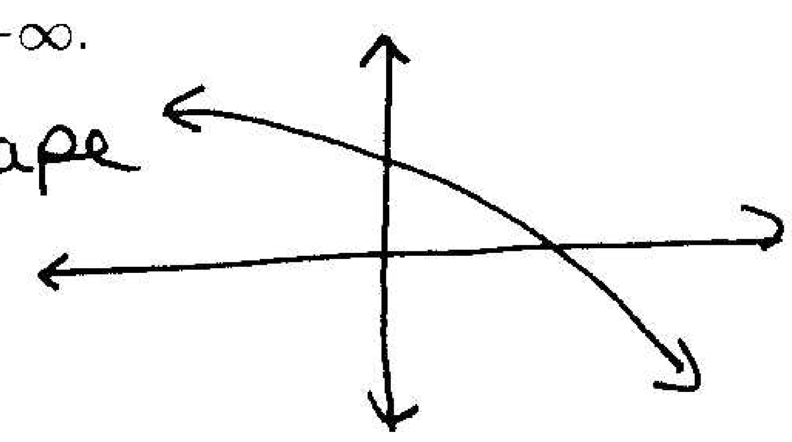


3 pts each

(d) If for all x, $f''(x) < 0$ and $f'(x) < 0$, then $\lim_{x\to\infty} f(x) = -\infty$.

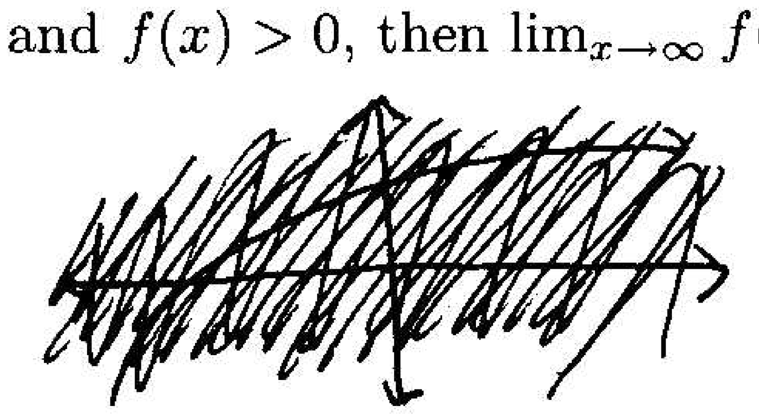
True must have this shape

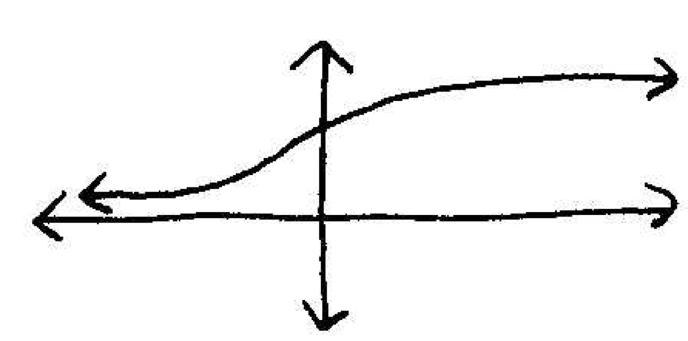
so $\lim_{x\to\infty} f(x) = -\infty$



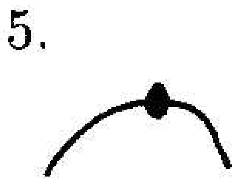
(e) If for all x, $f'(x) > 0$, and $f(x) > 0$, then $\lim_{x\to\infty} f(x) = \infty$.

False

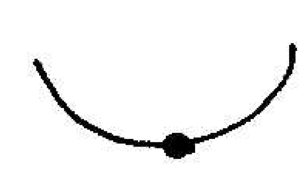




Given $f'(x)$ evaluated at 2, 4, and 5 is zero, and $f''(2) = -1$, $f''(4) = 0$, and $f''(5) = 2$, determine whether the point $(x, f(x))$ is a local minimum, local maximum, or neither for $x = 2, 4, 5$.

2   $2 \to f''(2) = -1$ concave down  maximum

1   $4 \to f''(4) = 0$ Neither

2   $5 \to f''(5) = 2$ concave up  minimum

Signature line: ______________________ ______________________