MATH 124 AND 125
FINAL EXAM REVIEW PACKET
ANSWERS

1.

t	1.0	1.2	1.4	1.6	1.8
$f'(t)$	1/2	3/4	5/4	7/4	2

2. __4___ $\lim_{h \to 0} \frac{f(20+h)-f(20)}{h}$

__1___ $f(16)$

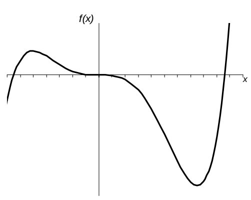
___2__ The slope of f at $x=10$

___3__ The average rate of change of f from $x=12$ to $x=24$

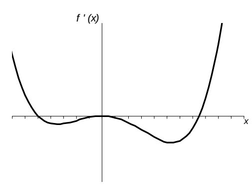
3. a) $g(-2)=3$, $g'(-2)=-1$ b) $g(-2)=-3$, $g'(-2)=1$

4. a) For $f(150)=125$: If a person weighs 150 pounds, the dose should be 125 milligrams.
For $f'(150)=3$: The dose for a 151 pound person would need to be approximately 3 milligrams higher than a dose for a 150 pound person.
b) 140 milligrams.

5. Graph of $f(x)$

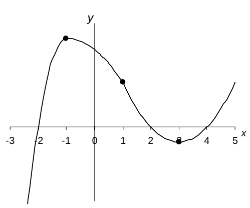


Graph of $f'(x)$



6. a) False b) True c) False d) False

7.



8. a) $\frac{1}{2}$ b) $\frac{1}{1+e}$ c) 0 d) 1 e) Does not exist.

9. $V(t)=25000(.85)^t$, $V'(3) \approx -2495.17$ dollars per year.

10. a) $\frac{dy}{dx}=\frac{1}{1+(a+x)^2}$ b) $\frac{dy}{dx}=\frac{-ax}{\left(a^2+x^2\right)^{3/2}}$

c) $\frac{dy}{dx}=-3a\cos^2(ax)\sin(ax)$ d) $\frac{dy}{dx}=-\ln a\left(a^{-x}\right)+ax^{a-1}$

11. Let $f(x)$ be a function so that $f(4)=3$ and $f'(4)=5$. Find the following:

a) $h'(4)=10$ $h'(x)=2f'(x)$ b) $g'(4)=-\frac{56}{9}$ $g'(x)=\frac{f(x)2x-x^2f'(x)}{\left(f(x)\right)^2}$

c) $k'(2)=20$ $k'(x)=f'(x^2)2x$ d) $m'(4)=-5e^{-3}$ $m'(x)=e^{-f(x)}(-1)f'(x)$

12. $x=5$, $x=-1$

13. a) $\frac{dm}{dv}=\frac{m_o v}{c^2\left(1-\left(v^2/c^2\right)\right)^{3/2}}$ b) $g'(x)=\begin{cases}2x & x<-3\\ -2x & -3<x<3\\ 2x & x>3\end{cases}$

Note: $g'(x)$ is not defined at $x=3$ and $x=-3$.

14. $f(h)=\frac{4}{5}h+20$

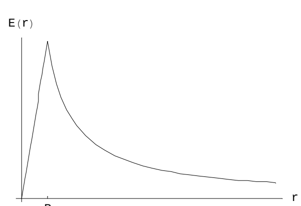
15. $k=15$

16. a) $\frac{3}{4}$ $2xy+x^2\frac{dy}{dx}+2\pi\sin\left(\pi x\right)=\frac{1}{x}-3y^2\frac{dy}{dx}$ b) $y=\frac{3}{4}x-\frac{7}{4}$

17. a) Yes. $E(R)=\lim_{r\to R^+}E(r)=kR$

b) No. The slope is *k* for $r<R$, but the slope approaches –*k* as *r* approaches *R* from the right.

c)



d) $\frac{dE}{dr}=\begin{cases}k & r<R\\ \frac{-kR^2}{r^2} & r>R\end{cases}$ Note: $\frac{dE}{dr}$ is not defined at $r=R$.

18. a) $t=3$ is a local minimum, $(3,-1/27)$
b) $t=4$ is the inflection point, $(4,-1/32)$
c) $t=2$ is the global maximum, $t=3$ is the global minimum

19. a) $\left(-a,2a^4+2a^3\right)$ is the local maximum, $\left(a,2a^4-2a^3\right)$ is the local minimum.
b) $\left(0,2a^4\right)$ is the inflection point.

20. a) $\lim_{t\to\pi}\frac{t^2-\pi^2}{\sin t}=-2\pi$ Use L'Hopital's rule.
b) $\lim_{\theta\to 0}\frac{\sin(2\theta)}{\sin(7\theta)}=\frac{2}{7}$ Use L'Hopital's rule.
c) $\lim_{x\to\infty}\arctan x=\pi/2$

21. $A=1/16$, $B=1/2$ Use $f(4)=1$ and $f'(4)=0$ to find *A* and *B*.

22. $a=0$, $b=4$, $k=3$

23. a) $c=-15$ b) $c=200$

24. a) $t=2$, $(-3,-16)$ Time when $\frac{dx}{dt}=0$ and $\frac{dy}{dt}=0$ b) $t=6$ Time when $y=0$.

25. The maximum volume is $\frac{L^3}{1728}$ cubic inches. The dimensions are $\frac{L}{12}\times\frac{L}{12}\times\frac{L}{12}$.
$V(x)=x\cdot x\cdot\left(\frac{L-x}{4}\right)$

26. The width is $\frac{10}{\sqrt{3}}$ and the height is $\frac{50}{3}$. $A(x)=(2x)\cdot(25-x^2)$

27. The minimum cost is $(9.6)V^{2/3}$ dollars. The dimensions are $(0.8)V^{1/3}\times(0.8)V^{1/3}\times\frac{V^{1/3}}{0.64}$.
$C(x)=5x^2+\frac{5.12V}{x}$

28. The wavelength is $w=c$ $V'(w)=\frac{k}{2}\left(\frac{w}{c}+\frac{c}{w}\right)^{-1/2}\left(\frac{1}{c}-\frac{c}{w^2}\right)$

29. The camera is rotating at $1/24$ radians per minute. $x = \tan\theta$, $\dfrac{d\theta}{dt} = -2.5$

30. The height is growing at $\dfrac{5}{432\pi}$ feet per min. $V = \dfrac{1}{3}\pi(3h)^2 h$. Differentiate with respect to *t*.

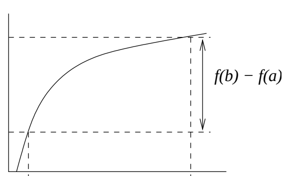
31. a) $\dfrac{b}{3}x^3 + x + c$ b) $b\ln|x| + \dfrac{1}{2}x^2 + c$ c) $\dfrac{1}{2}\ln\left|b + x^2\right| + c$ d) $\dfrac{1}{b}\arctan(bx) + c$

32. a) $\int_0^4 x(4-x)dx = \dfrac{32}{3}$ b) $\int_0^4 \left((x+2) - (x^2 - 3x + 2)\right)dx = \dfrac{32}{3}$

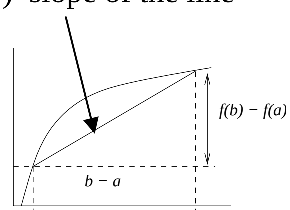
33. The lower estimate is 0.28. The upper estimate is 0.48.

34. a) Object B b) Objects A and D c) Objects A and D d) Object C

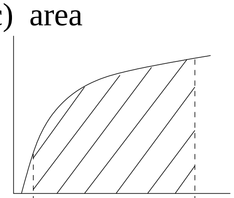
35. a) length



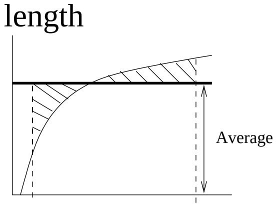
b) slope of the line



c) area

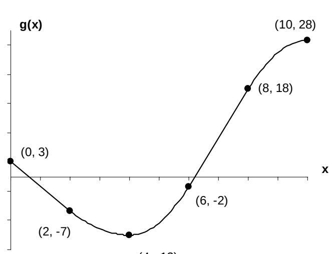


d) length

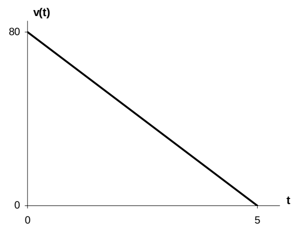


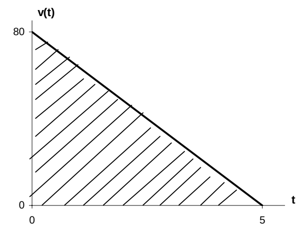
36. $\int_0^4 \left(3\sqrt{t} + 2\right)dt = 24$ liters.

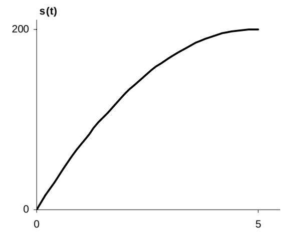
37.



38. a) $V(t) = -16t + 80$ b) 200 feet c) $s(t) = -8t^2 + 80t$







39. a) $F(0) = \int_0^0 e^{-t^2} dt = 0$ b) $F'(x) = \frac{d}{dx}\int_0^x e^{-t^2} dt = e^{-x^2}$

c) Increasing. d) Concave down. $F''(x) = -2xe^{-x^2}$

40. $\frac{1}{\pi/4 - 0}\int_0^{\pi/4} \frac{3}{\cos^2 x} dx = \frac{12}{\pi}$

41. a) $\int_{-\pi}^{\pi} \ln\left(5 + 4\cos x\right) dx = 4\pi \ln 2$ $f(x)$ is an even function.

b) $\int_0^{\pi/2} \ln\left(5 + 4\cos(2x)\right) dx = \pi \ln 2$ Let $u = 2x$ and change the endpoints.

42. a) True b) True c) False d) True e) False

43. a) $f'(x)$ is the graph that looks linear.

b) The local minimum is $x = -1$. The local maximum is $x = 5$.

$g'(x) = -f(x)e^{-x} + e^{-x} f'(x)$, $g'(x) = 0$ when $f(x) = f'(x)$.