

# MATH 251W: Foundations of Advanced Mathematics

## Spring 2011

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<b>Instructor:</b>	Erin McNicholas Office: Ford Hall, Room 211	email: emcnicho@willamette.edu Phone: 503-370-6590
<b>Office Hours:</b>	Monday 3-4:30pm Ford 211, Wednesday 9:30-11am Bistro	
<b>Math Tutoring:</b>	6:30-9:30 pm Sunday-Thursday in the Math Hearth	
<b>Class Website:</b>	Course information, assignments, due dates, and policies are all available on the course website under WISE	
<b>Class Meetings:</b>	Classes meet every Tuesday and Thursday. Section 01: 9:40-11:10 am in ETN 106, Final is Friday May 6th 8-11am Section 02: 12:50-2:20 pm in SML 130, Final is Saturday May 7th 2-5pm	

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*"In the broad light of day mathematicians check their equations and their proofs, leaving no stone unturned in their search for rigour. But at night, under the full moon, they dream, they float among the stars and wonder at the miracle of the heavens. They are inspired."*

- Michael Atiyah

### Class Objectives

To paraphrase G. H. Hardy, a mathematician is an artist working in the medium of ideas. In this class, you will learn how to use the tools of mathematics (namely conjecture, abstraction, and logic), to construct mathematical proofs. Proof writing is an integral part of all upper-level mathematics courses, and the method by which mathematicians extend the boundaries of what is known. It takes persistence, creativity, clarity, logical thinking, and an ability to see connections and patterns - skills which will help you in whatever field you pursue. Through this course you will: learn to read, understand, and compose mathematical proofs; distinguish valid arguments from invalid ones; formulate, write, and present logical arguments; develop your aesthetic sense in mathematics; and strengthen your analytic reasoning skills. Your grade will be based on your level of achievement in each of the following student learning outcomes:

- Your mathematical thinking<sup>1</sup>, proof reading, and proof writing  
As demonstrated on exams and homework sets
- Your familiarity with technological tools  
As demonstrated by your mastery of L<sup>A</sup>T<sub>E</sub>X
- Your participation in the mathematical community and your exposure to mathematical research  
As demonstrated by your attendance of mathematics colloquia and your collaboration with your peers

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<sup>1</sup>This includes your skill in abstraction, combining creative and analytical thinking, problem-solving, and formulating conjectures.

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## Required Course Materials:

*Proofs and Fundamentals: A First Course in Abstract Mathematics*, by Ethan Bloch  
A L<sup>A</sup>T<sub>E</sub>X-editor such as WinEdt (available in the Ford computer laboratories), MiKTeX, or LyX. For more information, see ‘LaTeX’ on the WISE site.

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## Course Components

**Exams (35%):** There will be two in-class midterms, each worth 10% of your course grade, and one cumulative final worth 15% of your course grade.

**Group Exams (30%):** There will be four group exams, each worth 10% of your course grade. Your lowest group exam score will be dropped. Group exams are done in groups of two or three, with each group member having a unique problem to work on. Group members proof-read each other’s work and offer suggestions. Of the 50 points possible for each group exam, 40 will be based on your work and 10 will be based on your proof-reading of team member’s exams. Each team member is allowed one page, one-sided, of notes. For more information on Group Exams, see the handout: Group Exams.

**Homework (30%):** Homework will consist of reading assignments, practice problems, and portfolio proofs (which must be generated using L<sup>A</sup>T<sub>E</sub>X). Your graded portfolio proofs will account for 30% of your course grade.

**Mathematical Development (5%):** Students are expected to attend 2 of the Math Colloquium Talks given throughout the semester. These talks will be announced in class and a schedule can be found on the Math Department web site [http://www.willamette.edu/~emcnicho/Math\\_Colloquium.htm](http://www.willamette.edu/~emcnicho/Math_Colloquium.htm). An excellent essay on how to approach math colloquium talks is available here: [http://www.willamette.edu/~emcnicho/courses/Multi249/How\\_to\\_listen\\_to\\_a\\_Math\\_Lecture\\_Korner.pdf](http://www.willamette.edu/~emcnicho/courses/Multi249/How_to_listen_to_a_Math_Lecture_Korner.pdf). The goal of this attendance requirement is to introduce students to the diversity and vitality of current mathematics research, and to include them in the Math Department culture.

**Grades:** Course grades will be based on a percent system. Grade cut-offs will be determined at the end of the semester. 90% and above will guarantee you an A-, 80% and above a B-, and 70% and above a C-.

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## Student Responsibility:

You are all adults and responsible for your own education. I will do everything in my power to help you learn. You should always feel free to stop by my office or make an appointment to meet with me. You should also feel free to ask me questions in class. Stop me if you are confused and ask me to explain things again. I welcome student questions! Although I will do everything in my power to help you through this class, you are ultimately responsible for your grade. The following is a list of things I expect from you.

- **READ THE TEXTBOOK.** This class is formatted under the assumption that you have completed the assigned reading before class. Class time will consist of lectures highlighting the main points of the section. It will be difficult to engage in the material during class if you have not looked over the content ahead of time. Furthermore, your textbook was chosen for its readability.
- **DO THE ASSIGNMENTS.** Mathematics is not a spectator sport. You will only learn mathematics by practicing, that is what homework is for. I encourage you to work with your fellow students on homework assignments. Make it a social activity and you will not only learn a great deal, you will have fun doing it. However, your L<sup>A</sup>T<sub>E</sub>X write-ups must be your own.
- **THINK CRITICALLY.** Your goal in this class should be to understand the concepts and strengthen your mathematical reasoning and proof writing skills. Throughout the course

you should be asking yourself “Why are we doing this? Why does this method work? How is this related to other topics I’ve learned? How would this apply to a concrete example?”

- **ASK QUESTIONS & SEEK HELP!** Ask questions in class, after class, during office hours, whenever! If you are confused or having problems with a certain section of the material see me **AS SOON AS POSSIBLE**. I am happy to help you but it is impossible to go over several weeks worth of material right before an exam.
- **STUDY.** The standard rule of thumb is that you should spend *three hours outside of class on course work for each hour spent in class*. To master the content of this course and to earn a good grade you will need to invest time and effort. Set aside time for both homework and studying.

### **Late Assignments and Missed Classes:**

I expect everyone to attend all classes and turn in all assignments. If for some reason you are unable to attend class or turn in an assignment, please let me know as soon as possible, preferably *before* the missed class or assignment. You should contact me prior to missing an exam. In most cases, if you have a valid excuse, the grade on the final will be used to replace your missed exam score.

### **Cell Phone/Screen Policy:**

No laptops, iPads, or other devices which take your eyes off your fellow classmates and the class discussion. Electronic devices such as cell phones, pagers, iPods, etc. must be turned off during class meetings. *If your cell phone goes off during class you will be responsible for bringing treats for the entire class at the next class meeting.* Papers should not be read during class, though I applaud your efforts to stay abreast of current events and tackle the latest crossword or sudoku puzzle.

### **Academic Integrity:**

In accordance with Willamette University CLA catalog: “Plagiarism and cheating are offenses against the integrity of the courses in which they occur and against the College community as a whole... Ignorance of what constitutes plagiarism shall not be considered a valid defense. If students are uncertain as to what constitutes plagiarism for a particular assignment, they should consult the instructor for clarification.” Cheating is unethical and I take it very seriously. The Deans Office will be notified if anyone is found cheating and appropriate sanctions will be given. If you are unsure of what constitutes cheating, please ask me.

## Tentative Schedule

The following schedule is subject to change.

Day	Class Discussions	Exams
Tu (1/18)	§1.1-2, Statements	
Th (1/20)	§1.3 L <sup>A</sup> T <sub>E</sub> X & Statements	
Tu (1/25)	§1.4, Arguments	
Th (1/27)	§1.5, Quantifiers	
Tu (2/1)	§2.1, 2.2 & 2.6, Direct Proofs	Group Exam 1
Th (2/3)	§2.3, Contrapositive & Contradiction	
Tu (2/8)	§2.4, Cases, IFF	
Th (2/10)	§2.5, Quantifiers in Proofs	NT Preview
Tu (2/15)	§3.1-2, Sets	Group Exam 2
Th (2/17)	§3.3-4, Set Operations	
Tu (2/22)	§4.1, Functions	
Th (2/24)	Midterm 1: Ch.s 1-3	
Tu (3/1)	§4.2, Image & Inverse Image	Analysis Preview
Th (3/3)	§4.3, Composition	
Tu (3/8)	No Class	
Th (3/10)	§4.4, Injective, Surjective	Applied Preview
Tu (3/15)	§5.1, Relations	Group Exam 3
Th (3/17)	§5.1, Relations & Groups	
3/21-3/25	Spring Break	
Tu (3/29)	§5.2, Congruence, Groups, & Fields	Abstract Preview
Th (3/31)	§5.3, Equivalence Relations	
Tu (4/5)	§6.1-2, Cardinality	
Th (4/7)	Midterm 2: Ch.s 4-5	
Tu (4/12)	§6.3, Induction	
Th (4/14)	§6.3-4, Induction & Recursion	Linear Preview
Tu (4/19)	§6.4, Recursion	
Th (4/21)	Ch. 0 §5.5-8.5, Sequences & Series	Group Exam 4
Tu (4/26)	Ch. 0 §5.5-8.5, Sequences & Series	Diff EQ Preview
Th (4/28)	Ch. 0 §5.5-8.5, Sequences & Series	
Tu (5/3)	Final Review	Topology Preview
F (5/6)	8-11am Final Section 01	
Sa (5/7)	2-5pm Final Section 02	

## From Previous Students to Future Students

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### On Colloquiums:

“I like having to go to these. They were interesting and it was neat to see what people are doing in math.”

“Extremely interesting”

“Going to colloquiums is fun!”

“Topics are not easy to get into, but widen my view of math.”

“I really liked these. They got me really excited about upper-level math.”

### Writing Proofs:

“Do scratch work, lay out the proof then attack. It may seem difficult or repetitive, but you can do it!”

“Little details are important. Often understanding comes in whether you know to put the small stuff into your proof.”

“It’s not all about math - there’s some flowery writing involved. Also make sure your terms are precise and concise.”

“Writing proofs is hard. Take advantage of office hours.”

“USE DEFINITIONS!!! That is how you do proofs. Always use whatever definitions exist to solve proofs. It makes life easier.”

“It takes a long time to write and make sure to leave a lot of time to do homework and spend a lot of time receiving help!”

### LaTeX:

“It’s frustrating at first, but stick with it and you’ll figure it out. It actually doesn’t suck after a month or so.”

“Have fun! Learn to look for little errors.”

“Make sure you use the available resources and the website.”

“Learn the symbols for it quickly! Also, use Google to search for topics when unknown.”

“Enclose anything that looks remotely mathematical in dollar signs, but exclude any words.”

“It takes a long time! Leave yourself a good amount of time to type things up.”

### Group Exams:

“Liked the ability to work with classmates to improve our understanding.”

“They were funnish, all with the exam buzz.”

“I like being able to work with others and get input on what I did before I turn it in.”

“Don’t blow them off! They may seem like they’ll be easier because you have people to help you, but they are harder questions than would be on the midterms.”

“Despite the fact that you work with others, you still are graded mostly on your own work. Don’t rely on your group to do it for you.”

“Prepare for a group exam just as you would for any other exam. The problems aren’t necessarily easier, but you have longer to do them; collaboration however can be difficult.”

“Do prepare and study for them, and prepare notes.”

“Be assertive - finish your problem before helping others. Don’t be scared to ask for extra quiet time.”

“Study and know theorems”

“Always prepare but don’t feel nervous. Group exams are a good way to learn.”

“Group exams help me learn during the exam while I do not need to be as nervous as taking a midterm.”

Midterms:

“Study everything! Go over all the homework again, even the proofs. Go over all the notes. Make notes when you are doing the homework on what seems important.”