This homework is based on the material presented in lecture; you should refer to on-line hand-outs and your own lecture notes for definitions, etc. Please ask for help if you have any questions.

1. **Syntactic abbreviations**

   Convert the following lambda terms into fully parenthesized form, with all multi-variable abstractions expanded. Let’s agree that “fully parenthesized” will mean that all applications and abstractions should be parenthesized, except for the outermost term (i.e., at top level).

   - $\lambda x y z. f x (f y y) (f z)$
   - $\lambda x y. f (\lambda x. x) (\lambda g f. g f x)$

   Perform the “opposite operation” on the following terms; i.e., use standard conventions on parenthesization and multiple abstractions to write them in a *minimal* form.

   - $\lambda a. (\lambda b. (a ((b b) a)))$
   - $(((\lambda p. (\lambda x. (p x)) (p a)) (\lambda q q)) (\lambda y. b)) a$

2. **Variables and binding**

   Convert the following terms so that no free or bound variables clash, i.e., so that all variables are distinct (respect the existing variable bindings: you may change the variables, but preserve the meaning).

   - $(\lambda x. (\lambda y. x (y x)) y) (\lambda y. (\lambda y. y x) y x)$
   - $b a (\lambda a b. a (\lambda a b) b) a (\lambda b. b a)$

3. **Substitution and reduction**

   Reduce the following lambda and combinator terms to normal form, using normal-order reduction, or argue that they have no normal form. Show intermediate $\beta$-reduction steps and, if necessary, variable renamings to avoid capture.

   - $(\lambda f g x. f (g x)) (\lambda y. y) (\lambda f x. f (f x)) x$
   - $(\lambda f x. f (f x)) (\lambda y. c (y y)) b$
   - $S I I (S I I)$