THE MARGINAL COSTS OF TAXATION AND REGULATION

by*

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* The positive attributes of this review are due entirely to the first two named authors. Any errors, omissions, or infelicities are the responsibility of the third named author and are the result of deadline pressures and ignorance.
THE MARGINAL COSTS OF TAXATION AND REGULATION

1. INTRODUCTION

Taxes and government regulations, which are the economic equivalents of government taxing and spending, impose a number of costs on a community. The most obvious cost is the revenue they raise or, in the case of regulations, the outlays they induce on the part of individuals and firms to comply with regulatory mandates. These losses are known as the direct costs of taxation or regulation. From the standpoint of the community as a whole, these costs are offset, to a greater or lesser extent, by benefits supported by taxation/regulation.

In addition to their direct costs, taxes/regulations impose additional resource costs on a community. These relate mainly to avoidance, evasion, compliance and administration (see Mikesell, this volume). In a free society individuals will arrange their affairs so as to minimize the amount of tax paid. Tax payments can be minimized legally by tax avoidance or illegally by tax evasion. In both cases significant resources of individuals, firms and specialist advisers are tied up in socially “unproductive” activities. Similarly, compliance with tax laws usually requires firms to keep additional records that they would not otherwise require. Enforcement and administration of the tax and regulatory systems also tie up significant amounts of society’s resources. This point is best summarized by Slemrod (1990) who notes:

> Taxation is a system of coercively collecting revenues from individuals who will tend to resist. The coercive nature of collecting taxes implies that the resource cost of implementing a tax system is large.

These losses are known as the enforcement cost of taxation or regulation.

A third cost of taxes and regulations arises from changes in behavior that they induce. These losses are known as deadweight costs or are sometimes referred to as the excess burden of taxation/regulation. Economists now pay more attention to the deadweight costs of taxes and regulation than they once did because deadweight costs are now evidently larger than they once were. Economists have always presumed that the deadweight costs of taxation, for example, could be minimized by imposing low marginal rates on good things like working or saving and investing, by broadening tax bases, by taxing bad things like pollution, tobacco, or alcohol, and by limiting the use of regulation.
to those situations where direct command is more efficacious than alternative governing instruments. However, minimizing the deadweight costs of taxation/regulation at negligible or very low levels is nearly impossible given the scope and scale of modern government.

1.1 How do deadweight costs arise?

Taxes and government regulations can distort incentives to work, save, and invest, and the pattern of input use and production in the economy. These distortions often impose costs on the economy by reallocating resources from more productive uses to less productive ones. Taxes, for example, distort incentives by placing ‘wedges’ between social and private returns. Presumably individuals allocate their labor and capital endowments to maximize returns (both monetary and non-monetary). To do this they must allocate their endowments so as to equalize after-tax returns in both high tax and low tax sectors. Low tax sectors include leisure, do-it-yourself work and the ‘shadow’ economy. Tax wedges induce people to allocate their resources to low tax sectors where the marginal social returns (as measured by the pre-tax return) are lower. Tax wedges thus divert resources from high tax to low tax sectors and act as barrier to raising total output (goods and leisure) that could result from reallocating resources.

As a specific example, consider taxes on income from labor. These taxes adversely affect incentives to work. When they increase, some people work fewer hours (i.e., they substitute away from work toward leisure); others work less intensively or undertake more do-it-yourself work; and a few shift into occupations offering relatively larger non-pecuniary benefits. The point is that in the absence of taxes people would have done things differently, which is to say that taxes have made them worse off, not only by the amount of the taxes they must pay, but also by causing them to shift away from their preferred patterns of work and leisure.

Tax wedges pervade the economy and it is their total impact that is important. Lindbeck (1986) argues the most obvious disincentive effects of marginal tax wedges can be summarized as substitution in favor of:

- leisure or recreation;
- lower intensity of work (‘on-the-job' leisure if wages are tied to productive effort);
- the pursuit of do-it-yourself work and production for barter;
- occupations with large non-pecuniary benefits; and
- the search for tax loopholes.
Some of the distortionary effects of tax wedges are less obvious. For instance, if the income tax system is progressive and the after-tax discount rate is relatively insensitive to tax increases, there will be an incentive to substitute away from investment in human capital (although in practice this may be offset to some extent by government subsidized education, see Stiefel and Schwartz, this volume). Labor mobility will also be affected by progressive taxes as wage differentials between regions or industries provide less incentive for people to incur the adjustment costs associated with changing locality or occupation.

Depending on the institutional and cultural characteristics on an economy, there may also be significant lags in the disincentive effects of taxation becoming fully apparent. For example, in economies in which wage and benefit increases are negotiated centrally, especially those committed to reducing wage differentials between men and women, occupations, firms, and regions (i.e., most European social democracies and, until very recently New Zealand and Australia as well, Scharph, 1991), the long-run effect of higher taxation is increased demands for tax free benefits such as shorter working hours or more liberal special-purpose leave conditions.

The structure of the tax system is also important. To the extent that the government relies on income rather than consumption taxes, people will save less due to the double taxation of savings inherent in most income tax systems. Investment choice can also be distorted when earnings from different kinds of assets are taxed at different rates. Consumption-related investments in housing, consumer durables, and collector's items are usually given favorable tax treatment relative to investments in stocks and bonds, bank deposits, and plant and equipment. Indeed, their implicit earnings are often not taxed at all (see, however, Foldvary, this volume). Inflation usually aggravates this situation in tax systems that are not indexed.

Given the highly mobile nature of capital, taxes and regulations that penalize investment in physical production assets or that make a country a less attractive place to invest relative to its competitors are likely to be particularly damaging. If capital accumulation suffers as a consequence, economic growth and living standards will be lower in the long-run.

1.2 The effects of taxation and regulation on economic growth

Many factors determine a country's rate of economic growth. The interactions between these factors are often complex. Typically, however, higher rates of investment in fixed and human capital lead to higher economic growth. Investments in institutional innovation
and the development of new products are also typically vital to the process. Insofar as high taxes reduce individual incentives to invest and to innovate and encourage people to substitute away from saving for future consumption in favor of current consumption, including increased leisure, they will adversely affect growth prospects.

A number of studies have attempted to examine the relationship between taxation/regulation levels and economic growth. While these studies typically lack any underlying analytical framework and rely instead on *ad hoc* regression analysis, they provide circumstantial evidence that high levels of taxation/regulation adversely affect economic growth. For instance, in a combined cross-section, time-series study of 103 countries between 1960 and 1980, Scully (1991) found that, on average, countries reached their maximum economic growth rates when they took less than 20 percent of GDP in taxes. Economic growth rates tended to zero as taxes increased beyond 20 percent and become negative when they consumed more than 45 percent of GDP.

Scully also found that governments maximized the dollar value of their revenue collections when 40-45 percent of GDP were collected in taxes. Attempts to take a larger share of private sector income actually led to reductions in the dollar value of revenue collected.

To illustrate the interaction of taxes and growth rates in his study, Scully compared two identical countries. If one set its tax rate to maximize current revenue (43.2 percent of GDP) while the other set its tax rate to maximize economic growth (19.3 percent of GDP), then after 40 years the country that maximized growth would have approximately the same government revenues as the high taxing country but its citizens would have more than three times as much after-tax income as the high tax country.

Littman reports (1990, 1991) reports the existence of a similar relationship between tax levels and economic growth within the United States: states with higher tax efforts (state taxes as a percent of the tax base) have been losing population to states with lower tax efforts. The ten states with the highest population growth between 1970 and 1990 had an average tax effort 12 percent below the national average. For the same period, the ten states with the lowest tax effort had an average employment growth of 18 percent while the ten states with the worst employment growth records all had tax efforts above the national average.

A recent study by Besci (1996) provides additional evidence that high state and local taxes adversely affect relative long-term growth. Besci notes that findings on growth effects of taxes have been mixed because empirical models imperfectly separated the growth effects of government actions that occur simultaneously with changes in tax rates.
His analysis shows that state and local spending have temporary growth effects that are stronger over shorter intervals than tax effects and permanent growth effects that do not disappear. His point is that public programs should not be adopted without considering costs, but neither should they be rejected without considering benefits.

Hahn & Hird (1991) estimated that the direct cost of federal regulation in the United States were between $275 billion and $350 billion per annum, or between $380 billion and $450 billion, if the incremental cost of new regulations enacted in 1990 were included. Hopkins’ (1991) total was a little higher. He estimated that the costs of federal regulation in the United States were between $430-562 billion per annum, about $6,000 per household. However, about $100 billion of this total relate to tax enforcement (see also Williams, Chen, and Tearney, 1989). The costs of disposing of hazardous wastes are excluded from both Hopkins’ and Hahn and Hird’s studies, as are the costs of endangered species protection, the minimum wage and other wages and hours legislation, the Americans with Disabilities Act, the Civil Rights Act of 1991, and price and entry controls in the public utilities, finance, insurance and real estate services, radio and TV broadcasting, and professional services industries. Other regulatory costs omitted from consideration by Hahn and Hird are the costs of the liability system administered through the courts (which peaked at $20.9 billion in 1987) and various mandatory no-fault compensation schemes such as workers' compensation (whose premiums totaled $31. These omissions are explained by Hopkins (1992, p. 8), who notes that estimates of the aggregate annual cost of federal regulation in the United States are "a patchwork quilt of studies and estimates with some important patches missing and others only partial coverage."

For the most part, general equilibrium analyses of regulation produce estimates that are multiples of direct compliance and enforcement costs. Jorgenson and Wilcoxen (1990) looked at the effects of environmental regulation on thirty-five industry groups and on the total American economy. They found that environmental regulations enacted prior to 1990 reduced the national product by about 2.6 percent and that the 1990 Clean Air Act would reduce it by a further 0.6 percent once its impact was complete. In a $7 trillion economy, 3.2 percent is $225 billion. A general equilibrium model of the United States constructed by Hazilla and Kopp (1990), produced similar results. Their model showed large and pervasive impacts from environmental regulation and significant intertemporal effects. Using a similar approach, Gray (1987) also identified significant intertemporal effects from social regulation. He concluded that OSHA and EPA regulation alone
reduced growth in the average manufacturing industry by .44 percent per annum. Of course, these estimates say nothing about the benefits of regulation.

1.3 Why is measuring deadweight costs important?

The deadweight cost measures the value of the opportunities that are lost when taxes or regulations divert labor, land and capital from their best uses. More precisely, they measure the extent to which the actual tax/regulatory system deviates from a ‘neutral’ system. A neutral tax system, for example, is one which leaves individuals’ decisions unchanged, relative to what they would have been if there were no tax system, but incomes are reduced by the precisely the amount of the revenue collected. A poll or head tax is the classic example of a neutral tax where people’s incomes are reduced by a given amount regardless of the actions they take. For most forms of taxation, however, the amount paid is influenced by the actions people take. Taxes cause people to adopt less preferred actions and, thereby, imposes additional costs on them over and above the amounts they remit to the taxman. One way of thinking about deadweight losses is that they are the maximum amount consumers and producers would pay to avoid taxes, less the revenue the taxes would have produced in the alternative case for the public fisc. This amount is almost always positive.

The size of deadweight losses is influenced by a range of factors but deadweight losses are likely to be greatest where the actions of producers and consumers are highly responsive to after-tax prices, where existing marginal tax rates are high and where savings are highly responsive to after-tax returns. Studies have typically found that the deadweight losses associated with raising taxation revenue range from a minimum of 10 cents for each additional dollar of revenue raised to well in excess of $1 for each additional dollar of revenue raised. Deadweight losses thus have the potential to be very significant indeed.

The calculation of deadweight losses is central to a number of policy questions including:

- which tax measures impose the least burdens or costs on the community to finance a public program or project?
- how valuable do public projects have to be to cover the full costs of the revenue needed to finance them? and
- how much redistribution from rich to poor can society afford?
The choice of the tax base/The design of regulatory instruments

Different methods of taxation have different deadweight costs. These costs can be central in choosing among methods of raising revenue. Thus one tax base may generate a deadweight cost of 20 cents for each additional dollar raised, while another may involve costs of 40 cents. In fact, these principles are often implicit in conventional views about public finance. The idea that a tax base should be as broad as possible has its origins in the observation that deadweight costs increase with higher tax rates, at an increasing rate. Thus the broader the base, the lower the tax rate and the lower the deadweight losses in aggregate. An optimal tax system would be one that equates the marginal deadweight losses across revenue sources. Similar considerations are relevant to the design of mechanisms to protect the environment, worker health and safety, etc. or to transfer income to politically favored groups such as farmers (see Friedman and Weare, this volume).

The value of public projects

Because additional government expenditures must be financed by raising taxes and because tax wedges generally distort choices of consumers and producers away from an efficient allocation of resources, the additional loss of efficiency due to increasing taxes or regulatory effort should be added to the monetary costs of additional government spending or abatement effort. In other words, the total cost to society of financing a marginal dollar of spending on behalf of a public purpose is the sum of that dollar (since it is diverted from another use) plus the deadweight costs of raising that dollar (see Lesser and Zerbe, this volume).

For example, study of the deadweight cost of taxing labor income in Australia estimated the marginal deadweight cost varied from 23 cents to as much as 65 cents (Findlay and Jones 1981). If correct, this means that an extra $1 of public expenditure costs at least $1.23 and, perhaps, as much as $1.65. The costs could be higher for other kinds of taxes. Deadweight costs are highest where behavioral responses to taxes are highest and the behavioral response of labor is relatively low on average.

The notion that a government project should earn a rate of return high enough to cover the excess burden created by the taxes used to pay for the project implies that, using Findlay and Jones’ figures, benefits should be at least 23 percent more than the value of the funds provided.
Cost-benefit calculations often neglect the cost of raising revenue and the true economic penalty — in terms of deadweight costs — is usually not even contemplated. A better understanding of the marginal deadweight cost of raising revenue and its careful application in cost-benefit assessments would help ensure value for money when spending the taxpayer’s dollar.

*The benefits from redistribution*

Raising revenue for transfer to the less well-off also involves deadweight costs. Even now the level of such transfers implicitly involves consideration of the cost to those from whom revenue is being taken. Because the costs of transfers also involve deadweight losses, these should be included.

Given that the taxation required to finance public expenditure is costly to raise, governments should leave money in the hands of taxpayers unless it is clear that they can do a better job of spending it. If the deadweight loss were 50 cents in the dollar, then reducing tax collections by a billion dollars would increase the purchasing power of the public by up to 1.5 billion dollars. Of course, considerations such as these may have the effect of increasing the relative attractiveness of transfers by regulation (the minimum wage, for example) (see Watts, this volume).

2. **MEASURING DEADWEIGHT COSTS:**

The behavioral changes caused by taxation and regulation can occur at a number of margins. Some of the most important are:
- willingness to work;
- choices among consumption goods;
- willingness to save;
- the pattern of savings;
- the production pattern in society;
- the use of inputs by particular industries; and
- the pattern of investment.

The measurement of the effects of taxation and regulation has received research attention in all these areas, with more or less success. The best of this work has been devoted to measuring the money value of the behavioral changes caused by the effect of taxes on labor supply. This case provides a convenient example for a diagrammatic exposition of the deadweight costs of taxation.
The measurement of the deadweight loss arising from the taxation of labor is illustrated in Figure 2.1. The willingness of workers to supply labor (say hours per week) at various hourly wage rates is indicated by $SS$, the compensated labor supply schedule. The demand for labor is denoted by $DD$. In Browning’s (1976) formulation of the excess burden concept (based on Harberger’s (1964) analysis of total excess burden), the demand curve was taken to be perfectly flat, which corresponds to a perfectly elastic demand for labor. In Figure 2.1, we extend Browning’s analysis to allow for a general demand curve for labor. In the absence of labor taxation, the equilibrium wage rate is $w_0$ and the equilibrium supply of labor is $L_0$. A tax at the rate of $t_1$ causes the wage received by workers to fall to $w_1(1 - t_1)$ and, at the lower wage, they are less willing to offer work. Labor supply consequently falls to $L_1$. The total loss of welfare to workers due to the imposition of the labor tax $t_1$ is the familiar deadweight loss triangle, $ABC$.

A more relevant concept than the total deadweight cost of taxation, however, is the marginal deadweight cost of taxation, since the interesting policy issue is not whether public spending should be abolished altogether, but whether public expenditure and related taxes should be raised, lowered or kept constant. Suppose we are considering adding an additional public sector spending program which will require an increase in the tax rate from $t_1$ to $t$ to be funded. This increase in taxation will lead to a further reduction
in labor supply to \( L(t) \) and the deadweight cost increases by the trapezoidal area \( BCFE \). If the additional public sector program is to be justified on efficiency grounds, then the benefits of the project should exceed the costs by at least \( BCFE \).

We can denote the incremental welfare loss \( BCFE \) as a function of the tax rate \( t \) by \( W(t) \). With linear supply and demand curves, it can be seen that the area defined by \( BCFE \) has the following analytic form:

\[
W(t) = \frac{1}{2} \left( \int_{L_1}^{L_1 t} w(t) \, dt \right) \left[ L_1 \right] L(t).
\]

We can denote the revenue raised by taxing labor income at the rate \( t \) by \( R(t) \). In Figure 2.1, \( R(t) \) is equal to the area of the rectangle joining the line segment \( EF \) to the \( w \) axis. Analytically, \( R(t) \) is defined as follows:

\[
R(t) = tw(t)L(t).
\]

The marginal excess burden associated with increasing the tax rate \( t \), \( MEB(t) \), where \( t = t_1 \), can be defined as the rate of change of the incremental excess burden, defined by (1), divided by the rate of change of the revenue, defined by (2); i.e., we have the following definition:

\[
MEB(t_1) = \frac{W(t_1)}{R(t_1)}
\]

where \( W(t_1) \) denotes the rate of change of \( W(t) \) with respect to \( t \) evaluated at \( t_1 \) and \( R(t_1) \) denotes the rate of change of \( R(t) \) with respect to \( t \) evaluated at \( t_1 \).

An explicit formula for \( MEB(t_1) \) in terms of demand and supply elasticities and the rate of labor taxation \( t_1 \) can be obtained if we approximate the inverse demand curve \( DD \) by the following linear approximation:

\[
w = w_1 - bL - L_1(t)
\]

where \( b \) is the slope of \( DD \) at the point \( B \). Similarly, we approximate the consumer’s inverse compensated labor supply curve \( SS \) by the following linear approximation:

\[
1 - t(w) = 1 - t_1(w_1 + c(L - L_1))
\]

where \( c \) is the slope of \( DD \) at the point \( C \). We can now treat (4) and (5) as two simultaneous equations and solve for \( w \) and \( L \) in terms of \( t \), obtaining the solution functions \( w(t) \) and \( L(t) \). Substituting these functions into (1) and (2) and evaluating the derivatives in (3) yields the following expression for the marginal excess burden evaluated at \( t = t_1 \):
Next we can define the negative of the elasticity of demand evaluated at the point B as \( \frac{w}{L} \). Hence, the elasticity \( \frac{w}{L} \) and the slope \( b \) are related as follows:

\[
(7) \quad b = \frac{w}{L}. \]

We can define the compensated elasticity of supply evaluated at the point C as \( \frac{1}{h} \). Hence, the elasticity \( \frac{1}{h} \) and the slope \( c \) are related as follows:

\[
(8) \quad c = \left(1 \cdot \frac{t}{1}ight) \frac{w}{1} / \frac{1}{h} L. \]

Substitution of (7) and (8) into (6) yields the following expression for the marginal excess burden:

\[
(9) \quad MEB(t_1) = \frac{t}{\left[\left(1 \cdot \frac{t}{1}\right)\frac{1}{h} + \left(1 \cdot \frac{1}{h}\right) t\right]}.
\]

\[
(10) \quad = \frac{1}{h} t_1 / \left[\frac{1}{h} + \left(1 \cdot \frac{1}{h}\right) t_1\right].
\]

where (10) follows from (9) if \( \frac{1}{h} \) and \( \frac{w}{L} \) are both non-zero and finite.

The case considered by Browning (1976) (and corrected by Findlay and Jones (1982; 556)) is a special case of (9) when \( \frac{1}{h} = 0 \). In this case, (9) reduces to:

\[
(11) \quad MEB_{J}(t_1) = \frac{t_1}{1 \cdot \frac{1}{h} t_1}.
\]

which in turn is approximately equal to Browning’s (1987; 13) amended formula for the marginal excess burden:

\[
(12) \quad MEB_{B}(t_1) = \frac{t_1}{1 \cdot \frac{1}{h} t_1}.
\]

The above diagram and analysis illustrates the Harberger-Browning partial equilibrium approach to measuring the incremental excess burden that can be associated with an increase in taxes to finance a government project. Note that this approach leads to the rather complex formulae (9) or (10) when the demand for labor function is not perfectly elastic. Note also that if either \( \frac{1}{h} \) (the supply elasticity) or \( \frac{w}{L} \) (the negative of the demand elasticity) are zero, then the marginal excess burden will also be zero.

By differentiating the right hand side of (10) with respect to \( t_1, \frac{1}{h} \) and \( \frac{w}{L} \), it can be shown that the marginal excess burden increases as \( t_1 \) (the tax rate on labor income), \( \frac{1}{h} \) (the supply elasticity) and \( \frac{w}{L} \) (the negative of the demand elasticity) increase.
The above partial equilibrium approach to defining marginal excess burdens has a number of important limitations: (i) the approach is limited to changes in labor tax rates and it is not clear how to extend the approach to changes in other tax rates; (ii) the change in $t$ may induce changes in other prices and quantities which may affect welfare; (iii) the partial equilibrium approach does not specify precisely what the government will do with any extra tax revenue; and, (iv) it is not clear whether consumers receive transfer payments from the government to keep them at a constant utility level as tax rates are varied. The above difficulties (and additional ones) were raised by Stuart (1984), Ballard, Shoven and Whalley (1985), Hansson and Stuart (1985), Ballard (1990) and Fullerton (1991). The general response of the above authors to the problems raised by the partial equilibrium approach has been to specify a small general equilibrium model of an economy with given explicit consumer, producer and government budget constraints. The incremental disincentive effects of raising any government tax rate is then evaluated in the context of the general equilibrium model specified.

2.1 Key studies of the magnitude of deadweight costs

As we noted at the start of this Chapter, Browning (1976) wrote the seminal paper in the literature on deadweight costs of taxation. He formalized the notion of the marginal cost of public funds and applied his methodology to calculate the deadweight cost for taxes that affect labor in the United States.

Browning used the standard deadweight formula developed by Harberger (1964). The formula shows that the total deadweight cost aggregated over all workers is:

\[
W = \frac{1}{2} \partial t^2 Y
\]

where $W$ is the deadweight cost, $\partial$ is the elasticity of labor supply (compensated for income effects) with respect to a change in disposable income, $t$ is the tax rate and $Y$ is total labor income.

To use Browning’s measure we need an estimate of the elasticity of labor supply adjusted for income effects. The logic behind this intuition is as follows. The imposition of a tax on labor income reduces the incentive to work relative to consuming leisure. However, it also reduces disposable income and this has a separate affect on the amount of consumption of goods and leisure that an individual chooses. This income effect does not normally affect allocative choices and should, therefore, be separated out when measuring deadweight costs. (Another reason for separating out income effects is that our counterfactual is
usually assumed to be a lump sum tax, which would have income effects but no substitution effects.

Browning examined the effect of a change in the tax rate by taking the derivative as follows:

$$dW = \int tY dt$$  

where $dW$ is the change in the deadweight cost and $dt$ is the change in the tax rate.

Assuming a proportional income tax, revenue ($R$) is given by:

$$R = tY$$

and additional revenue for a change in the tax rate (for an unchanged tax base) is given by:

$$dR = Y dt.$$

The marginal welfare or deadweight cost per dollar of revenue raised is therefore:

$$\frac{dW}{dR} = \frac{\int t}{t}.$$

The marginal cost of one dollar of public funds is the marginal welfare cost of taxation plus the direct cost or $(\int t + 1)$.

The above formulae were derived for the case of a proportional tax. Browning also considered the case of a flat rate tax with an exemption up to a certain limit (known as a degressive tax) and of a tax with graduated rates that increase with incomes (known as a progressive tax).

The deadweight cost for a degressive tax is given by:

$$\frac{dW}{dR_D} = \int t \frac{Y}{B}$$

where $B$ is the tax base. Since $Y$ is greater than the tax base, the deadweight cost for a degressive tax will be greater than for a proportional tax. Browning shows that with an exemption of 40 percent of average income, $Y / B$ would be 1.6. In order to raise the same revenue as a proportional tax, the marginal rate under a degressive tax would have to rise from 35 to 56 percent. This implies that the marginal deadweight cost for a degressive tax would be 2.5 times larger than for a proportional tax yielding the same revenue.

The deadweight cost for a progressive tax varies depending on how the different rates in each tax bracket are varied. The general formula is as follows:

$$\frac{dW}{dR_P} = \int i \int t_i Y_i + B_i * \text{change in } t_i + \text{change in } t$$
where $t$ is the average tax rate for all brackets and there are $i$ brackets. Browning used formulae 5, 6 and 7 to calculate the deadweight costs of raising an additional dollar in taxation after taking account of the existing marginal tax rates implied by the US federal, state and local income and sales taxes, payroll taxes and social security taxes. He thus assumed that all these taxes affect the decision to work and are effectively borne fully by labor. For example, sales taxes reduce the purchasing power value of earnings and this induces a substitution from taxed goods to untaxed goods including leisure.

Browning's results are shown in Table 2.1. They show that the marginal deadweight cost of raising additional tax revenue varies from 8 percent to 16 percent of the additional revenue depending on whether a proportional or a progressive tax structure is used. Although Browning’s work was path breaking, his estimates of the magnitude of marginal deadweight costs are low compared to subsequent, more sophisticated studies.

As noted above, Browning (1987) modified his approach to adjust for an error arising from the fact that data relating to a situation in the presence of a tax were used, whereas the formula applied to the situation in the absence of a tax. This understated the welfare cost by a factor of $1/(1-t)$. The revised estimates of the marginal deadweight costs as a percent of tax revenue varied from 8 to well over 100 percent.

Findlay and Jones (1982) identified this error in Browning's original paper. They also allowed the tax base to vary in their methodology, which they applied to Australian data for income, excise, and sales taxes. For a compensated elasticity of supply of 0.2 they found the deadweight cost varied from 23 to 65 percent of tax revenue, depending on whether the rate structure change was proportional, degressive or progressive. For an elasticity range of 0.1 to 0.4, they found a deadweight cost of 11 to 160 percent of tax revenue.

A number of other approaches to measuring the deadweight costs of taxation have been developed in the literature. Differences can often be traced to the definition of consumer utility and the specification of labor supply, and whether partial or general equilibrium approaches are used.
Stuart (1984) and Ballard, Shoven and Whalley (1985) derived their measures from the computation of a two sector and a multi-sector general equilibrium model, respectively. Their results illustrate some key general equilibrium implications.

As noted by Stuart (1984), Browning's approach is strictly only valid for small changes and, more importantly, it compares an undistorted equilibrium to a fully compensated situation. However, most tax changes start with a distorted equilibrium and lead to a new distorted equilibrium. Another problem is that the equilibrium level of welfare and tax revenue depends on the way the government spends the revenue. Stuart overcomes these problems and also relaxes Browning's assumption of a fixed tax base by using a simple general equilibrium framework.

Stuart’s model assumes two sectors, corresponding to a taxed market sector and a non-taxed household (and leisure) sector. Capital stocks in each sector are assumed fixed and immobile between sectors. Simple, explicit production and utility functions are assumed. Government expenditure takes two forms; consumption which does not affect utility, and transfers that increase household income. Stuart’s model yields a general equilibrium computation for the deadweight cost of taxation, based on United States data on personal income, payroll, and excise taxes -- since all these can be avoided if labor shifts from taxed to untaxed uses. He estimates that deadweight costs varied from 21 to 100 percent as a percent of tax revenue for compensated elasticities of supply from 0.2 to 0.84 and a marginal tax rate of 42.7 percent. Their range was 24 to 133 percent for marginal tax rates of 46 percent.

The foregoing estimates are based on the assumption that all marginal tax revenue is distributed on a lump sum basis. When the alternative assumption is made that all tax revenue is used to finance government consumption, the benchmark deadweight cost falls from 21 to 7 percent. The intuition of this result is that when revenue is directed to government consumption, individuals receive no income (or consumption) benefits and they do not consume more leisure. But when the revenue is spent on transfers, the income effects mean that they do consume more leisure making it more difficult to raise revenue. Thus, in the former case it is easier to raise revenue because of income effects and this lowers the deadweight cost burden. This striking result suggests that the marginal excess burden of government expenditure, even wasteful government expenditure is less than for redistributional programs.

Ballard, Shoven and Whalley (1985) used a multi-sector, intertemporal, computational general equilibrium model to calculate simultaneously the welfare effects of all major taxes
in the United States. They estimated the marginal deadweight loss from a 1 percent increase in all distortionary taxes. For a plausible range of elasticities they found deadweight costs amount to from 17 to 56 percent of revenue raised (Table 2.2). They also demonstrated that the deadweight cost is higher when elasticities and tax rates are larger.

Table 2.2: Marginal Deadweight Cost Estimates of Ballard, Shoven and Whalley

<table>
<thead>
<tr>
<th>Labor supply elasticity (uncompensated)</th>
<th>Saving elasticity</th>
<th>percent of revenue</th>
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<tbody>
<tr>
<td></td>
<td>0.0</td>
<td>0.4</td>
</tr>
<tr>
<td>0.0</td>
<td>17</td>
<td>21</td>
</tr>
<tr>
<td>0.15</td>
<td>27</td>
<td>33</td>
</tr>
<tr>
<td>0.30</td>
<td>39</td>
<td>48</td>
</tr>
</tbody>
</table>

Source: Ballard, Shoven and Whalley (1985)

Jorgenson and Yun (1990) also used a multi-sector general equilibrium model of the United States to calculate the welfare effects of the complete tax system. Their model differed from that of Ballard, Shoven and Whalley by specifying a very detailed representation of the tax system. In particular, distinctions were made between short- and long-lived assets and among assets held in the corporate, non-corporate and household sectors. Distinctions were also made between average and marginal tax rates and the different tax treatment of different types of income.

For the tax system after the 1986 tax reform, they found a marginal efficiency cost of 38 percent of tax revenue and an average efficiency cost of 18 percent of tax revenue.

They also calculated efficiency costs before the tax reform and compared them with earlier estimates (Table 2.3). They found generally higher estimates, especially for capital. It should be noted that they model capital taxes in much more detail than Ballard, Shoven and Whalley and therefore show inter-asset and inter-sectoral capital distortions to be very large. This is consistent with strong substitution possibilities among capital assets.

Table 2.3: Comparison of Marginal Deadweight Cost Estimates

<table>
<thead>
<tr>
<th></th>
<th>Ballard, Shoven and Whalley</th>
<th>Jorgenson and Yun</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital</td>
<td>46</td>
<td>92</td>
</tr>
<tr>
<td>Labor</td>
<td>23</td>
<td>48</td>
</tr>
<tr>
<td>All</td>
<td>33</td>
<td>46</td>
</tr>
</tbody>
</table>

Source: Ballard, Shoven and Whalley (1985), Jorgenson and Yun (1990)
These studies illustrate that the deadweight costs of taxation can be very high, even for relatively low-tax countries such as the United States. The few studies which have been carried out for the high-tax, European 'welfare state' countries indicate that deadweight costs can become massive when very high taxes are combined with complex social welfare systems. For instance, using a two sector, two input general equilibrium model, Hansson and Stuart (1985; 345) found that the marginal cost of public funds in Sweden was around $2.30 for government spending on transfer payments and $1.70 for government spending on goods and services for each additional dollar of taxes collected. Marginal tax rates in Sweden at the time were around 70 percent for the average taxpayer (inclusive of all taxes and income-dependent transfers).

Most calculations of marginal excess burdens are subject to some severe limitations. Even where an applied general equilibrium model is used, most rely on restrictive functional forms for producer’s production functions and consumer’s preference functions. The three most commonly used functional forms in applied general equilibrium theory are the Cobb-Douglas, constant elasticity of substitution and Leontief (no substitution) functional forms. If the number of goods in the model is greater than two, these functional forms impose severe a priori restriction on elasticities of substitution; e.g., strict complementarity is ruled out; see Diewert (1985a).1 Finally, with the salient exceptions of the applied general equilibrium models of Jorgenson and Yun (1986a) (1986b) (1990) (1991), the elasticity estimates that are used in most empirical work are often taken from studies pertaining to other countries.

We take up the general equilibrium approach in the next section of this review. However, instead of using restrictive functional forms to model consumer and producer behavior or relying on guesstimates for the relevant elasticities, we suggest that it is possible to estimate statistically these elasticities using flexible functional form techniques.

3. ALTERNATIVE MODELS OF MARGINAL EXCESS BURDEN

In the previous section of this review, we gave a diagrammatic and algebraic exposition of the partial equilibrium approach to measuring the marginal excess burden of a tax and expenditure increase and discussed some general equilibrium approaches.2

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1 For an excellent survey of applied general equilibrium modelling, see Shoven and Whalley (1984).

Browning (1987) suggested that most of the uncertainty surrounding estimates of the welfare costs of government taxation/regulation results from uncertainty about the magnitude of various elasticities of supply and demand. Stuart (1984), Ballard (1990) and Fullerton (1991) have all argued, however, that an equally important source of differences in estimates of marginal excess burdens is to be found in differences in assumptions. Thus in this section, we set out the alternative assumptions that could underlie a simplified general equilibrium model of an economy in some detail and the alternative concepts of the marginal excess burden of a tax increase implied by each.

In section 3.1, we start with a hypothetical planned economy where an optimal allocation of resources can be attained without taxes. This model is not presented for its realism, but to introduce assumptions about consumers and producers and to show what a first best allocation of resources would look like. In section 3.2, we introduce taxes and a decentralized market economy. Our first conception of marginal excess burden is introduced here. In section 3.3, we introduce our second concept of marginal excess burden which follows the example of Kay and Keen (1988) and uses a variant of Debreu’s (1951) (1954) coefficient of resource utilization to measure the excess burden of a tax increase. In this second concept, as a tax rate increases, consumers are given an offsetting transfer which keeps them at the same level of real income. Thus, our second concept of marginal excess burden can be viewed as a rigorous general equilibrium specification of the original Harberger (1964) – Browning (1976) marginal excess burden measure. Section 3.4 concludes with a nonmathematical summary of this discussion.

### 3.1 The Optimal Allocation of Resources

Consider a simple model of a closed economy. There are three goods in the economy: (i) a consumption good $C$; (ii) labor $L$ or leisure $h = H - L$ (where $H$ is total hours potentially available for work in the period under consideration) and (iii) a fixed factor $K$ (an aggregate of land and capital). There are three sectors in the economy: (i) a household sector that demands consumer goods and supplies labor; (ii) a private production sector that produces a composite good that is consumed both by consumers and the government and uses labor $L_P$ as an input and (iii) a government sector that consumes goods $G$ and uses the amount of labor $L_G$ to produce general government services.

---

3 Debreu’s work was preceded by that of Allais (1943) (1977). The loss measures of Allais and Debreu were put in a unified framework by Diewert (1983) (1984).
The technology of the private production sector can be represented by a production function $f$ where

\[ C + G = f(L_P, K). \]

$C + G$ is the total output produced given that $L_P$ units of labor are used.

The preferences of the household sector are represented by the utility function $U$. The utility level achieved $u$ depends on consumption $C$ and leisure $h$ where

\[ u = U(C, h); \]
\[ h = H - L_P - L_G, \]

so that total labor supply is $L = L_P + L_G$. The maximum level of welfare that is achievable in this economy can be obtained by maximizing utility, $U(C, H - L_P - L_G)$, subject to the production function constraint, $C + G = f(L_P, K)$, with respect to consumption $C$ and privately utilized labor supply $L_P$. The government requirements for goods $G$ and labor $L_G$ are held fixed. Upon substituting the production function constraint into the utility function, our welfare maximization problem reduces to:

\[ \max_{L_P} U(f(L_P, K) - G, H - L_G - L_P). \]

The first order necessary condition for solving (4) is:

\[ U_C f_L + U_h (\Pi) = 0. \]

Thus at the optimal solution, we will have

\[ f_L(L_P, K) = U_h(C^*, h^*) / U_C(C^*, h^*). \]

Equation (6) implies that the slope of the production function will equal the slope of the consumer’s indifference curve at the optimum. The geometry of problem (4) is illustrated by Figure 3.1. In Figure 3.1, the distance $OA$ is equal to $L_G$, the government's labor requirements. The total output that can be produced by the economy is the curve $AB$, the production function constraint. The distance $FE = AC$ is the government’s goods requirement $G$. The curve $CD$ is $AB$ shifted down by $G$. The indifference curve $II$ is the highest one that is tangent to $CD$ and thus the consumer’s equilibrium point is $E$ and the producer’s equilibrium point is at $F$. Note that the slope of the line tangent to $F$ is equal to the slope of the line tangent to $E$; this is the geometry behind condition (6) above.
Of course, in real life economies, the government expenditures on goods, $G$, and on labor, $L_G$, must be financed by taxes or user charges. Thus in the following section, we introduce taxes into the above model.

### 3.2 Marginal excess burdens: A first approach

In order to model an equilibrium distorted by taxes, we need to introduce the following tax rates: $t_1$ is the rate of taxation on consumption goods, $t_2$ is the rate of taxation on labor and $t_3$ is the rate of taxation on the fixed factor. We denote the producer price for the consumption good by $p_1$, and the producer price for labor by $p_2$. The consumer prices for these two goods are $p_1(1 + t_1)$ and $p_2(1 - t_2)$ respectively.

In order to obtain the equations which characterize a tax distorted equilibrium, it is convenient to use duality theory.\(^4\) Thus we assume that the expenditure function dual to the utility function $U(C, h)$ is $\ell_p\{1 + t_1\} - p_2\{1 + t_2\} - u$ and the profit function dual to

\(^4\) For expositions of duality theory, see Diewert (1974) (1993; Ch. 6) or Varian (1984; Ch. 1 and 3).
the production function \( f(L_p, K) \) is \( L[p_1, p_2, K] = L[p_1, p_2] \) where we have dropped \( K \) since it is held fixed.

The equations which define a tax distorted equilibrium are (7) – (10) below.

(7) \[ e_1[p_1(1+t_1), p_2(1 \cdot t_2), u] = \square_1(p_1, p_2) \square G \]

(8) \[ e_2[p_1(1+t_1), p_2(1 \cdot t_2), u] = \square_2(p_1, p_2) + H \square L_G \]

(9) \[ e[p_1(1+t_1), p_2(1 \cdot t_2), u] = (1 \cdot t_3) \square (p_1, p_2) + p_2(1 \cdot t_2)H \]

(10) \[ t_1 t_2 e[p_1(1+t_1), p_2(1 \cdot t_2), u] + t_2 p_2 H \square e_2[p_1(1+t_1), p_2(1 \cdot t_2), u] \]

\[ + t_3 \square (p_1, p_2) = p_1 G + p_2 L_G \]

Equations (7) and (8) are the demand equals supply equations for goods and labor, respectively; (9) is the household budget constraint and (10) is the government’s budget constraint. Differentiation of a function with respect to its \( i \)th variable is denoted by a subscript \( i \).

As is usual in general equilibrium theory, not all four equations (7) — (10) are independent. Hence we drop equation (10). We also require a normalization on prices. We choose the producer price of labor \( P_2 \) to be the numeraire good and hence, we have

(11) \[ P_2 = 1. \]

Equations (7) — (10) can now be regarded as 3 simultaneous equations in 6 unknowns: \( u \) (the level of household utility), \( G \) (the level of government expenditure), \( P_2 \) (the price of the consumer good), \( t_1 \) (the tax rate on consumer goods), \( t_2 \) (the tax rate on labor) and \( t_3 \) (the tax rate on capital). Then if we \( u, G, \) and \( P_2 \) as endogenous variables and the tax rates \( t_1, t_2, \) and \( t_3 \) as exogenous variables, equations (7) — (9) determine the functions \( u(t_1, t_2, t_3) \), \( G(t_1, t_2, t_3) \), and \( p_1(t_1, t_2, t_3) \).

In order to determine the overall effects of a tax increase, we have to aggregate the consumer’s change in utility with the change in government real expenditures. One way of aggregating utility \( u \) and real government expenditures on goods \( G \) is by assuming the
existence of an overall social welfare function that is additive in the two components.\(^5\) Thus, a money metric\(^6\) welfare indicator \(W\) can be defined as follows:

\[
W(u, G, P_1, P_2) \equiv e(P_1, P_2, u) + P_1 G + P_2 L\]

where \(P_1 \equiv [1 + t_1]p_1\) and \(P_2 \equiv [1 - t_2]p_2\) are reference consumer prices for the consumption good and labor (or leisure), respectively. Thus, we measure the overall welfare of the representative consumer by private (per capita) expenditures on goods and leisure, plus (per capita) government expenditures on goods, \(e(P_1, P_2, u)\), plus (per capita) government expenditures on labor, \(P_2 L\), where all expenditures are evaluated at the reference \(P_1\) and \(P_2\). As utility \(u\) and government expenditures \(G\) are changed due to the change in the tax rates \(t_1, t_2,\) or \(t_3\), we hold the reference prices \(P_1\) and \(P_2\) constant in (12).

To determine welfare as a function of the tax rates \(t_1, t_2,\) or \(t_3\), we substitute our solution functions \(u(t_1, t_2, t_3)\) and \(G(t_1, t_2, t_3)\) to the system of equations (7) — (9) into (12) to obtain the welfare function \(W(t_1, t_2, t_3)\):

\[
W^*(t_1, t_2, t_3) \equiv e(P_1, P_2, u(t_1, t_2, t_3)) + P_1 G(t_1, t_2, t_3) + P_2 L\]

To calculate the marginal excess burden of a tax increase in \(t_i\), \(EB_i\), we simply calculate minus the change in welfare due to the increase in \(t_i\) and divide by the change in real government spending on goods; i.e., \(EB_i\) is defined as follows for \(i=1,2,3\):

\[
EB_i = \left[ e_u(P_1, P_2, u(t_1, t_2, t_3)) + P_1 u_i(t_1, t_2, t_3) + P_2 L_i(t_1, t_2, t_3) \right] / G_i(t_1, t_2, t_3)
\]

where \(e_u\) denotes the partial derivative of \(e(P_1, P_2, u)\) with respect to \(u\), \(G_i\) denotes the partial derivative of \(G(t_1, t_2, t_3)\) with respect to \(t_i\) and \(u_i\) denotes the partial derivative of \(G(t_1, t_2, t_3)\) with respect to \(t_i\) for \(i=1,2,3\). Note that \(EB_2\) defined by (14) is the general equilibrium counterpart to the partial equilibrium excess burden measure \(MEB(t_i)\) defined earlier by equation (3) in section 2 above.

The marginal excess burden measures, \(EB_i\) defined by (14 is illustrated in Figure 3.2. In Figure 3.2, the initial producer equilibrium point is at the point \(F\) on the private sector.

\(^5\) This type of assumption was used by Atkinson and Stern (1974; 174).

\(^6\) The term money metric scaling is due to Samuelson (1974; 1262).
production possibilities frontier $AB$. The government’s labor requirements are $OA$ and the government’s initial goods requirements $G_1$ are $FE$. The initial consumer equilibrium is at the point $E$ which is on the indifference curve $I_1I_1$ which corresponds to the initial utility level $u_1$. The tax rate $t_1$ on the consumer good is increased. This shifts the consumer’s budget line down and the new consumer equilibrium is at the point $K$ on the indifference curve $I_2I_2$ which corresponds to the lower utility level $u_2$. The new producer equilibrium is at the point $J$ and the government’s new consumption of goods $G_2$ is the distance $JK$. Measuring utility in terms of the consumption good, it can be seen that the decrease in utility is equal to the distance $EH$. From the diagram, the distance $JK$ is less than the distance $FH$. This corresponds to the inequality $G_2 < G_1 + u_1$ or $u_2 + G_2 < u_1 + G_1$. Thus, as the tax rate on consumption goods $t_1$ increases, government spending $G$ increases but utility $u$ decreases and overall welfare measured in units of the consumer good, $u+G$, decreases. This welfare decrease divided by the government expenditure increase corresponds to the marginal excess burden measure $EB_1$ defined by (14).

Figure 3.2: Marginal excess burden
In the remainder of this section, we summarize the computation of the partial derivatives on the right hand side of (15), which defines the excess burden measures \( E_b_i \). To calculate the partial derivatives \( u_{it_1}, t_1, t_2, t_3 \) and \( G_{it_1}, t_2, t_3 \), it is necessary to differentiate equations (7) — (9) above with respect to the endogenous variables \( G, u, \) and \( P_1 \) and the exogenous tax variables \( t_1, t_2, \) and \( t_3 \). Using matrix notation, the resulting system of equations is:

\[
\begin{bmatrix}
  e_{11} & e_{12} & 0 & dt_1 \\
  e_{21} & e_{22} & 0 & dt_2 \\
  e_1 & e_2 & 0 & dt_3
\end{bmatrix}
\]

where \( e_i = \partial e(P_1, P_2, u) / \partial P_i \) and \( e_{ij} = \partial e(P_1, P_2, u) / \partial P_i \partial P_j \) are the first and second order partial derivatives of the expenditure function \( e(P_1, P_2, u) \) with respect to the consumer prices \( P_i \) and \( P_j \) for \( i, j = 1, 2 \); \( e_{iu} = \partial e(P_1, P_2, u) / \partial P_i \partial u \) is the cross partial derivative with respect to \( P_i \) and \( u \) for \( i = 1, 2 \) and \( \Box_i = \partial \Box(P_1, P_2) / \partial P_i \) and \( \Box_{ij} = \partial \Box(P_1, P_2) / \partial P_i \partial P_j \) are first and second order partial derivatives of the producer’s profit function \( \Box(P_1, P_2) \) with respect to the producer prices \( P_i \) and \( P_j \) for \( i, j = 1, 2 \). From duality theory, the first derivatives of the profit and expenditure functions with respect to prices are equal to net supply functions and (Hicksian) consumer demand functions, respectively.

Thus in order to calculate the excess burden measures \( E_b_i \), we need only use the last two equations in (16) to calculate the derivatives \( \partial p_i / \partial t_i \) for \( i = 1, 2, 3 \). If increasing tax rate \( t_i \) decreases the producer price of output (relative to the wage rate which we are holding constant), then \( E_b_i \) will have the expected positive sign; i.e., increasing \( t_i \) to finance additional government expenditures will lead to a decrease in the producer price of output which in turn will lead to a drop in output and utility.

---

7 The full computations are rather long and complex. The reader who is interested in the details of the computation should refer to Diewert and Lawrence, 1994.

8 See Diewert (1974; 112 and 137) (1993; 131 and 166) for example.
Using the last two equations in (16) to calculate the response of the output price \( P_1 \) to changes in the tax rates \( t_i \) leads to the following equations:

\[
\frac{\partial p_1(t_1, t_2, t_3)}{\partial t_1} = \frac{e_{21} u}{D} C e_{2u} + \frac{e_{22} u}{D} L e_{2u} ;
\]

\[
\frac{\partial p_1(t_1, t_2, t_3)}{\partial t_2} = \frac{e_{22} u}{D} P_1 C e_{2u} + \frac{e_{21} u}{D} P_2 L e_{2u} ;
\]

\[
\frac{\partial p_1(t_1, t_2, t_3)}{\partial t_3} = \frac{e_{2u} u}{D} H e_{2u} + \frac{e_{21} u}{D} C e_{2u}.
\]

where total labor supply is \( L = L_p + L_G \) and \( D \) is defined as:

\[
D = e_{2u} u \left[ C \left( 1 + t_1 \right) \right] \left[ \left( 1 + t_2 \right) \left( C + G \right) \right] \left[ e_{21} u \left( 1 + t_1 \right) \right] \left[ e_{21} u \left( 1 + t_1 \right) \right] .
\]

It is interesting to note the key role played by \( e_{2u} u \equiv \frac{\partial e_{2u}}{\partial P_1, P_2, u} \), which is the response of leisure demand to an increase in real income \( u \). If \( e_{2u} \) is sufficiently large and positive and \( D \) remains negative, we get the rather anomalous results \( \frac{\partial p_1}{\partial t_1} > 0 \), \( \frac{\partial p_1}{\partial t_2} > 0 \), and \( \frac{\partial p_1}{\partial t_3} > 0 \), which means that overall welfare will actually increase as we increase \( t_1, t_2 \), and \( t_3 \). Assuming also that \( \frac{\partial G}{\partial t_i} \) is positive for each \( i \), we find under the above conditions that the marginal excess burden measures \( EBi \) defined by (14) become negative so that there is an excess benefit instead of an excess burden associated with the tax increase. The explanation for this rather result is that the tax increase leads to a large fall in real income or utility, which leads to a large drop in the demand for leisure, a large increase in the supply of labor, and a more than offsetting increase in output, i.e., the increase in \( G \) outweighs the decline in \( u \).

This section can be summarized as follows: the marginal excess burden measures \( EBi \) associated with an increase in the tax rate \( t_i \) are defined by (15). The derivatives in the numerators of (15) can be expressed in terms of initial tax rates \( t_i \), the initial allocation of resources (\( C, h, L \) and \( G \)), the responses of (Hicksian) consumer demands to changes in prices \( e_{ij} \), and the responses of producer net supply functions to changes in prices \( \varpi_{ij} \).

---

9 See Hicks (1946), Samuelson (1947 & 1953) and Diewert (1993 & 1974).

10 \( D \) is the determinant of the two by two submatrix involving \( du \) and \( dp_1 \) of the matrix on the left hand side of (16).

11 Our intuitive explanation for the existence of marginal excess benefits follows that of Fullerton (1991; 305). For some values of the parameters in their applied general equilibrium models, Hansson and Stuart (1985; 333) and Ballard (1990; 269) found negative marginal excess burdens or positive excess benefits.
using equations (17) — (20). Similar formulae for the derivatives of (15) can be obtained. The resulting formulae for the marginal excess burdens are unfortunately rather complex.

In addition to complexity, there is another major difficulty with this approach to measuring excess burdens. The difficulty is that our method for measuring total welfare $W$ defined by (12) by summing together consumer expenditures on private goods with expenditures by governments on goods is completely arbitrary. This problem becomes acute in a many-consumer context owing to the lack of a universally accepted metric for aggregating changes in private utility as a result of changes in government expenditures to obtain a measure of overall welfare change. In the next section we outline an approach to measuring excess burdens that circumvents this measurement problem.

3.3 Marginal excess burdens: A second approach

Our second approach to measuring the excess burden of a tax increase is based on the approach to efficiency measurement pioneered by Allais (1943) (1977) and Debreu (1951) (1954). In order to avoid the problem of adding together a utility change with a change in public output, we hold each consumer’s utility constant as tax rates are increased.

In the context of our representative consumer model described in 3.2, utility $u$ is held constant by adding a transfer payment $T$ to the consumer’s income. The endogenous variables in our simple general equilibrium model become $G$ (government expenditures on goods), $T$ (the transfer), and $P_1$ (the producer price of output). The numeraire good is again labor and the producer price of labor $P_2$ is held constant. The exogenous variables are $t_1$, $t_2$, and $t_3$, the tax rates on consumption, labor earnings and profits, respectively. The new system of equations which describes our model is given by (7) and (8) (the demand equals supply equations for goods and labor) and equations (21) and (22) below:

\begin{align}
(21) & \quad e[p_1(1 + t_1), p_2(1 + t_2)]u = (1 + t_3)\square(p_1, p_2) + p_2(1 + t_2)H + T; \\
(22) & \quad t_1p_1\square[p_1(1 + t_1), p_2(1 + t_2)]u + t_2p_2p_2\square[p_1(1 + t_1), p_2(1 + t_2)]u \\
& \quad + t_3\square(p_1, p_2) = p_1G + p_2L_G + T
\end{align}

12 This point is made rather effectively in Kay and Keen (1988; 268).

Equation (21) is the consumer’s budget constraint and (22) is the government budget constraint.

As is usual in general equilibrium theory, the four equations (7), (8), (21) and (22) are dependent. We drop (22) and use the remaining equations to solve for $G$, $T$, and $P_1$ as functions of the tax rates $t_1$, $t_2$, and $t_3$. The partial derivatives of these solution functions can be obtained by totally differentiating (7), (8) and (21) with respect to $G$, $T$, $P_1$, and $t_1$, $t_2$, and $t_3$. Using matrix notation, the resulting equations may be written as follows:

\[
\begin{bmatrix}
1 & 0 & e_{11}(1 + t_1) & 0 \\
0 & 1 & 0 & e_{21}(1 + t_1) \\
0 & 0 & C(1 + t_1) & 0 \\
0 & 0 & -1 & C(1 + t_1)
\end{bmatrix}
\begin{bmatrix}
\frac{dG}{dt_1} \\
\frac{dT}{dt_2} \\
\frac{dp_1}{dt_3}
\end{bmatrix}
= \begin{bmatrix}
e_{11}P_1 & e_{12}P_2 & 0 \\
e_{21}P_1 & e_{22}P_2 & 0 \\
Cp_1 & Lp_2 & 0
\end{bmatrix}
\begin{bmatrix}
dt_1 \\
dt_2 \\
dt_3
\end{bmatrix}
\]

where (17) is used in evaluating the derivatives in (23).

Because utility remains constant in this model, any benefits generated by an increase in taxation are just equal to the change in government purchases of goods $G$, valued at the initial consumer price of goods $P_1$. Thus our indicator of overall welfare in the present model is simply

\[
(24)
W(t_1, t_2, t_3) = P_1 G(t_1, t_2, t_3)
\]

where $G(t_1, t_2, t_3)$ (along with $T(t_1, t_2, t_3)$, ..., and $p_1(t_1, t_2, t_3)$) are the functions obtained by solving (7), (8) and (21). From Equation (22), we see that the government revenue raised, $R$, is equal to government expenditures on goods, $P_1G$, and labor, $P_2L_G$, plus government transfers to consumers, $T$. Thus we can define tax revenue as a function of the tax rates $t_1$, $t_2$, and $t_3$ as follows:

\[
(25)
R(t_1, t_2, t_3) = P_1 G(t_1, t_2, t_3) + P_2 L_G + T(t_1, t_2, t_3).
\]

Our general equilibrium measure of the marginal excess burden associated with increasing the tax rate $t_i$, $MEB_i$, can now be defined as (minus) the rate of change in welfare defined
by (24) divided by the rate of change in revenue defined by (25) with respect to $t_i$; i.e., for $i = 1, 2, 3$:

$$MEB_i = \left[ \frac{\partial W(t_1, t_2, t_3)}{\partial t_i} \right] \left[ \frac{\partial R(t_1, t_2, t_3)}{\partial t_i} \right]^{-1} = \frac{W(t_1, t_2, t_3)}{R(t_1, t_2, t_3)}$$

where $W_i, R_i, G_i, p_{1i}$ and $T_i$ are the partial derivatives of the functions $W(t_1, t_2, t_3), \ldots , T(t_1, t_2, t_3)$ with respect to $t_i$ for $i = 1, 2, 3$. There will be an excess burden associated with increasing $t_i$ if $MEB_i$ is positive, an excess benefit if $MEB_i$ is negative.

The derivatives $G_i(t_1, t_2, t_3)$ that appear in (27) can be obtained by inverting the matrix on the left hand side of (23), which yields:

$$MEB_3 = 0.$$  

In order to obtain explicit expressions for $MEB_1$ and $MEB_2$ in terms of tax rates and various supply and demand elasticities, we need to calculate the transfer derivatives $T_1(t_1, t_2, t_3)$ and $T_2(t_1, t_2, t_3)$. This can readily be done using the third equation in (23). The resulting partial derivatives are:

$$T_1(t_1, t_2, t_3) = p_1 C + \left[ \left( t_1 + t_3 \right) C \left( 1 \left[ \begin{array}{c} t_1 \\ t_2 \\ t_3 \end{array} \right] \right) G \right] \frac{\partial p_1(t_1, t_2, t_3)}{\partial t_1} ;$$

$$T_2(t_1, t_2, t_3) = p_2 L + \left[ \left( t_1 + t_3 \right) C \left( 1 \left[ \begin{array}{c} t_1 \\ t_2 \\ t_3 \end{array} \right] \right) G \right] \frac{\partial p_2(t_1, t_2, t_3)}{\partial t_2} .$$

Finally, (29) and (30) may be substituted into (27) in order to obtain formulae for the Allais-Debreu excess burden measures $MEB_1$ and $MEB_2$ in terms of the initial allocation of resources, the initial tax rates and the responses of net demand and supply functions to changes in prices (the $e_{ij}$ and $\varpi_{ij}$). The resulting formulae are too complex to be presented.

---

14 This equation tells us that the Allais-Debreu excess burden of a tax increase on profits is zero. This result follows from the model’s assumptions, which hold that capital is a fixed factor that is unaffected by a tax on its use. However, this result does not extend to a dynamic model where reproducible capital is endogenously determined. Thus the result (29) should not be used in the design of a real life tax system. Theoretical and empirical research indicates that the efficiency costs of taxing the return to capital can be very high: see Ballard, Shoven and Whalley (1985), Jorgenson and Yun (1986a) (1986b) (1990) (1991), and Diewert (1988; 23).
here. Instead, we will present results for an approximation to the general formulae (27) that will be accurate for low tax rates \( t_i \) and government expenditures on goods \( G \), which leads to the following approximate Allais-Debreu excess burden measures, \( MEB_i^* \).

\[
MEB_1^* = \left( t_1 + t_2 \right) \frac{\Omega_{12} s \Omega_{12}}{\Omega_{12} s \Omega_{12} + \Omega_{12} (1 + t_1)} \geq 0 ;
\]

\[
MEB_2^* = \left( t_1 + t_2 \right) \frac{\Omega_{12} s \Omega_{12}}{\Omega_{12} s \Omega_{12} + \Omega_{12} (1 + t_1)} \geq 0
\]

where \( s_C \equiv P_1 C / P_1 Y \) and \( s_L \equiv P_2 L / P_1 Y \) are the consumption and labor shares of output valued at consumer prices, \( \Omega_{12} \equiv e_{12} P_2 / Y \geq 0 \) is the cross elasticity of demand for consumption with respect to leisure, and \( \Omega_{12} \equiv - e_{12} P_2 / Y \geq 0 \) minus the cross elasticity of supply of output with respect to labor. A comparison of (31) and (32) shows that:

\[
MEB_2^* = \left( \frac{S_C}{S_L} \right) MEB_1^* .
\]

Thus, if the consumption share of output, \( s_C \), is greater than labor’s share of output (valued at consumer prices), then the (approximate) marginal excess burden associated with raising labor taxes will exceed the burden associated with raising consumption taxes.

Examination of (31) and (32) shows that for normal parameter values, (approximate) marginal excess burdens will increase as the tax rates \( t_1 \) on consumption and \( t_2 \) on labor earnings increase and as substitutability in consumption and production increase (i.e., as \( h_{12} \) and \( s_{12} \) increase).\(^{15}\) Note also that \( MEB_1^* \) and \( MEB_2^* \) will equal zero if either \( h_{12} \) or \( s_{12} \) equals zero. Hence to get positive excess burdens in our simple general equilibrium model, we must have strict substitutability in both production and consumption. This is similar to the situation that occurred in the partial equilibrium model developed in section 2.\(^{16}\) Finally, note that the excess burdens defined by (31) and (32) are approximately proportional to \( t_1 + t_2 \), the sum of the tax rates on consumption and labor earnings.

The geometry of the numerators of the marginal excess burden measures defined by (26) can be illustrated using Figure 3.2 again. If the initial producer equilibrium were at point \( F \) and the initial consumer equilibrium at point \( H \) and the initial government consumption of goods the distance \( FH \), increasing the tax rate on consumption \( t_1 \) or the tax rate on labor \( t_2 \) is increased would cause the consumer price line to shift (become less steeply sloped) relative to the producer price line. Consequently, producers would move down the

\(^{15}\) These theoretical results are broadly consistent with the results obtained in the applied general equilibrium models of Stuart (1984; 360) and Ballard, Shoven and Whalley (1985; 128).

\(^{16}\) Compare the partial equilibrium formula for the marginal excess burden of a labor tax given by (10) in section 2 with (32) in the present Section.
production possibilities set $AB$ to point $J$ and consumers move along the indifference curve $I_1I_2$ to point $K$. Government consumption would then be $JK$, which is less than initial government consumption $FH$.

### 3.4 Summary of the section

Our goal in this chapter has been to outline excess burden measures that are valid in a general equilibrium context that can be used as substitutes for the standard elasticity measures of marginal burden based on simple partial equilibrium models.

Our first general equilibrium excess burden measure was based on the assumption that society’s objective function (or social welfare function) was equal to a constant dollar sum of private household consumption and leisure plus the constant dollar sum of government expenditures on goods and services. However, the resulting measure proved both overly complex and excessively arbitrary: there is no reason why the benefits of government expenditures should be precisely additive to the consumer’s constant dollar consumption of goods and leisure.

Our second general equilibrium excess burden measure was, therefore, based on the assumption that consumers’ money metric utility over private goods remained constant when taxes are raised to finance increased government on goods, holding government expenditures on labor constant. Since private utility and government expenditures on labor were held constant, this measure of social welfare became government expenditures on goods, at constant reference prices: see (24) in 3.3.

An intuitive example of the logic behind this second model of marginal excess burden can be explained as follows: Taxes are increased and the increased revenues are used to increase government. However, the increased tax wedge causes increased deadweight loss and, in particular, a decrease in private utility for consumers. To restore consumers to their pretax levels of private utility, the government provides consumers with a tax transfer. It turns out that this tax transfer *more than exhausts* the increase in revenue that the initial tax increase created. Thus the governmental activity financed by the initial tax increase should be valued by consumers by enough of a premium to overcome the effects of the increased loss of efficiency generated by the initial tax increase. This premium rate is the estimated marginal deadweight loss.
4. AN ILLUSTRATION: THE CASE OF NEW ZEALAND

In this section we show how the general equilibrium approach outlined in the last section of this review can be exploited to estimate marginal deadweight losses from taxation/regulation using flexible functional form techniques (i.e., techniques that do not impose unwarranted a priori restrictions on elasticities of substitution between the outputs and inputs). The example we use is that of New Zealand. The New Zealand economy is of considerable interest. It has undergone extensive reform in the last decade. Reform of the tax system has been an integral part of this process. More reliance has been placed on indirect taxes with the introduction of one of the most comprehensive goods and services taxes in the world, the income tax has been made broader-based but with a flatter rate structure, and import tariffs have been scaled down.

However, tax revenue as a proportion of GDP has continued to increase and is high relative to comparable countries. In 1991, it was 38.2 percent compared with 29.9 percent in the United States and 30.8 percent in Australia. New Zealand’s tax share in 1991 was also higher than that of Germany, the United Kingdom and Japan (see Figure 4.1). While all these countries’ tax shares have increased over the last 25 years, New Zealand’s tax share has increased more rapidly (although it has fallen somewhat since its peak in 1990 and is projected to fall further). Furthermore, the tax shares of OECD countries tend to be high compared to the dynamic Asian economies. For instance, South Korea, Singapore, Thailand and Indonesia had tax shares of less than 17 percent in 1991.

Figure 4.1: Tax Revenue as a Percentage of GDP — Selected OECD Countries
Moreover, government expenditure in New Zealand have exceeded tax revenues in eleven of the last 12 years producing substantial increases in the public debt. In 1992-93 its public debt was 55 percent of GDP (Richardson 1992). High levels of government spending and debt in the present necessarily imply high taxes in the future.

Estimation of the deadweight costs of taxation using a comprehensive modeling framework requires extensive time-series information on a range of key economic variables. The construction of consistent time-series database for the period 1971-72 to 1990-91 for the New Zealand economy to permit detailed econometric work to be carried out is reported in Diewert and Lawrence (1994).

The general equilibrium approach outlined in the last section of this review involves three components: a model of consumer behavior, a model of producer behavior, and the marginal excess burden model itself. Econometric estimation of the models of consumer and producer behavior requires consistent time-series information the following variables as well as on tax payments classified by 7 major categories.

The static consumer model estimated requires value, price and quantity information on 4 consumption categories:

- housing;
- motor vehicles;
- general consumption (excluding transport and housing); and,
- leisure.

The additional data series required for the future estimation of a dynamic or intertemporal consumer model relate to the components of household wealth (other than housing and motor vehicles), foreign debt and discount rates.

The production model estimated required value, price and quantity information on the following output and input categories:

- Outputs: motor vehicles;
- general consumption excluding housing and transport (private and public);
- housing investment;
- general investment:
  - non-residential and other construction investment;
— plant, machinery and transport equipment investment;
— changes in inventories (agricultural and non-agricultural);
- exports;
- Variable Inputs: • imports;
  • labor;
- Fixed Inputs: • capital:
  — non-residential and other construction stocks;
  — plant, machinery and transport equipment stocks;
  — inventory stocks (agricultural and non-agricultural);
and,
  • land (excluding government holdings).

In constructing the databases for the consumer and producer models it is necessary to specify the series in terms of consumer and producer prices, respectively. This requires detailed information on the magnitude and composition of tax payments and government subsidies which form the wedges between prices paid by the consumer and those received by the producer or supplier.

An important distinction which arises in all econometric studies of this type is the difference between stocks and flows. Most outputs from the production sector and some of the inputs to it are produced and consumed in the one period. This makes their measurement relatively easy. However, many of the inputs used in the production process and many of the major consumption items are durable assets and last several periods (or decades in some cases). Measuring the amount of these durable items consumed in any one period is problematic and requires measurement of the flow of services provided by the asset over its lifetime. Measurement of the stock, or total value of the asset held is also not straightforward due to the presence of inflation and alternative assumptions about depreciation rates and must be constructed using economic conventions rather than the national accounts.

Exports and imports also raise problems. One way to deal with these problems is to treat consumption, exports and imports as separate goods. This approach is not conventional but it has appeared in the literature in the past 15 years: see the articles by Kohli (1978) (1993) and Lawrence (1989). Of course, some goods are both consumed and exported. However, the transportation, marketing and storage of these “identical” goods serves to
differentiate them from each other; e.g. export margins will generally be different from domestic margins for the same good. Moreover, these are aggregate measures representing thousands of goods in each of the categories, “imports,” “exports,” and “consumption.”. The relative proportions of each micro good in these aggregates are different and hence the price indexes for each of these aggregates will be quite different, even if each aggregate were composed of a different mix of exactly the same goods. Thus, the only practical way to deal with these aggregation difficulties is to treat “export” and “consumption” as separate goods. If they are in fact virtually the same, then this fact will show up as extremely high substitutability between the two goods in econometric work. The same logic applies to imports: virtually all imports have domestic inputs added to them in terms of transportation, storage, packaging, wholesaling and retailing inputs. As a matter of national income accounting conventions, imports do not simply disappear: after domestic value added has been added to them, they reappear as components of consumption. Finally, New Zealand is a small country. This means the prices of New Zealand’s export and imports are set on foreign markets and can be treated as exogenous.

4.1 Producer behavior

In what follows, \( x = (x_1, x_2, x_3, x_4, x_5) \) denotes a vector of variable net outputs for the New Zealand economy, \( p = (p_1, p_2, p_3, p_4, p_5) \) is the corresponding positive vector of variable input and output prices that producers face, \( s = (s_1, s_2) \) is a vector of stocks that are available to producers at the beginning of the year under consideration and \( w = (w_1, w_2) \) is a vector of ex post rental prices associated with the stocks. Technology is represented by a GNP function (or variable profit function), \( (P, s, t) \), defined as follows:

\[
\max_x \left\{ p \cdot x : (x, s) \text{ belongs to } S^t \right\}
\]

The functional form for used here is a variation of the normalized quadratic functional form, since this functional form allows the imposition of appropriate curvature conditions without destroying its flexibility properties. Using matrix notation, this function can be defined as follows:

\[
\max \left\{ p \cdot Cs + p \cdot ch \cdot st + p \cdot gd \cdot st \right\}
\]

---

\[ + \left( \frac{1}{2} \right) p \cdot A \cdot h \cdot s / p \cdot g \left( \frac{1}{2} \right) s \cdot B \cdot p \cdot g / h \cdot s \]

where the vectors \( g = (g_1, g_2, g_3, g_4, g_5) \) and \( h = (h_1, h_2) \) were chosen a priori to be the absolute values of the sample means of the observed \( x^t = (x_1^t, ..., x_5^t) \) and
\[
w^t = (w_1^t, ..., w_5^t),
\]
normalized so that:
\[ P^* g = 1; s^* h = 1 \]

where the \( P^* \) and \( s^* \) were fixed vectors. The variable \( t \) which appears in (2) is a scalar time variable which serves as a proxy for technological change.

With the above statistical specification, non-linear maximum likelihood programs such as SHAZAM (see White (1978)) can be used to estimate the unknown parameters that appear in (2), after dropping any one of the transformed estimating equations. The resulting estimates will be invariant to the equation dropped. This treatment is completely consistent with standard techniques used in the theory of demand, where data are also subject to an adding up constraint.

Diewert and Lawrence (1994) used the non-linear regression program in SHAZAM to estimate the unknown coefficients appearing in (2) using New Zealand data. In their analysis the resulting measure of technical progress (obtained by differentiating the profit function with respect to \( t \) and dividing by \( (P, s, t) \) evaluated at \( P = P^2 \) and \( s = s^2 \)) turned out to be 0.061 for 1972 and trended upward to 0.106 for 1991, averaging 8.2 percent per year for the twenty years in their sample. Since variable profits were only about one quarter of the total returns to labor and capital, the average rate of 8.2 percent translates into an average total factor productivity improvement of about 2 percent per year. Their analysis further indicated that this technical progress was mainly due to export augmenting and labor saving.

Because the fitted net output of variable good \( i \) in period \( t \), \( x_i^t \), can be obtained by differentiating \( (P^b, s^b, t^b) \) with respect to \( P^b \), the \( j \)th price elasticity of net supply for good \( i \) in period \( t \) can be defined as
\[ \left[ j \right]_{ij} = \left[ p_{ij}^t \right] / x_i^t \left[ \partial^2 \left( p_{i}^{t}, s_{i}^{t}, t \right) / \partial p_{i} \partial p_{j} \right] ; \ i, j = 1, ..., 5. \]

---

18 We chose \( P^* \) and \( s^* \) to be vectors of ones.
19 Recall equation (2) above; thus we have for.
The sample means of the net supply elasticities are listed in Table 4.1. From viewing Table 4.1, it can be seen that with the exception of the price elasticity of demand for labor (which averaged -0.47), the elasticities were rather small in magnitude. However, there were some interesting trends in the annual elasticity estimates: the own price elasticity of supply for motor vehicles trended upwards from 0.06 in 1972 to 0.12 in 1991; the own price elasticity of supply for general output trended up from 0.23 to 0.36; the own price elasticity of supply for exports stayed approximately constant at 0.16; the own price elasticity of demand for imports stayed approximately constant at -0.30 and then trended to -0.24 during the last 5 years, and the own price elasticity of demand for labor trended up in magnitude from -0.35 in 1971 to -0.71 in 1991.20 From our discussion in section 3.3, the increasing magnitudes of the own price elasticity of supply for general output and the own price elasticity of demand for labor suggests that the excess burden of increased government spending in New Zealand was probably be increasing over time. As we shall see in section 4.3, this expectation of increasing excess burdens turns out to be true.

### 4.2 Consumer behavior

In estimating a model of consumer demand responsiveness (i.e., a system of consumer demand and labor supply equations) it is necessary to have value, price, and quantity data on each of the main items consumed.

Diewert and Lawrence (1994) assumed that a representative consumer had preferences defined over 4 current period goods: (1) the consumption of the services of the stock of motor vehicles; (2) general consumption (excluding motor vehicles and housing); (3) the consumption of the services of the current stock of housing; and (4) the consumption of leisure. The economy’s total consumption of the above 4 goods was divided by the adult,

---

20 We have found this same upward trend in the magnitude of the price elasticity of demand for labour for most OECD countries in similar production models. This suggests that increases in wage rates have led to greater rates of unemployment in these countries.
working age population, aged 15-64 inclusive. Each working age adult was given a time endowment of 2000 hours per year. Per capita leisure \( h \) was defined as 2000-\( L \) where \( L \) is per capita hours of work supplied during the year under consideration.

In this approach the representative consumer’s preferences are expressed by the expenditure function, \( e(u, p) \), which is dual to the utility function, \( u = f(x) \), where \( p \) and \( x \) are price and quantity vectors pertaining to consumer expenditure categories.\(^{21}\) As in the previous section, a normalized quadratic functional form is used here,\(^{22}\) since curvature conditions can be imposed on this functional form without destroying its flexibility. The functional form used in this section is defined as follows:

\[
e(u, p) = \begin{cases} 
a \cdot p + b \cdot pu + \left( \frac{1}{2} \right) p \cdot Cpu / p \cdot g & \text{for } u \leq u^* \\
 a \cdot p + b \cdot pu^* + c \cdot p(u^* u^*) + \left( \frac{1}{2} \right) p \cdot Cpu / p \cdot g & \text{for } u > u^* 
\end{cases}
\]

where \( g \equiv (g_1, g_2, g_3, g_4) \) is a predetermined parameter vector; \( u^* \) is a predetermined level of utility; \( a \equiv (a_1, a_2, a_3, a_4) \), \( b \equiv (b_1, b_2, b_3, b_4) \), and \( c \equiv (c_1, c_2, c_3, c_4) \) are parameter vectors to be estimated and \( C \equiv c_{ij} \) is a symmetric parameter matrix to be determined. The parameter vectors \( a, b, \) and \( c \) satisfy the following restrictions:

\[
a \cdot p^* = 0; \quad b \cdot p^* = 1; \quad c \cdot p = 1
\]

where \( p^* \) is a predetermined price vector. The parameter matrix \( C \) satisfies the following restrictions:

\[
C = -U^T U
\]

where \( U = \begin{bmatrix} u_{ij} \end{bmatrix} \) is an upper triangular matrix which satisfies the following restrictions:

\[
Up^* = 0_d.
\]

\(^{21}\) For expositions of the use of duality theory in modelling consumer preferences, see Diewert (1974; 120-133) (1993; 148-154).

The restrictions (2) — (4) impose money metric scaling\(^{23}\) on the representative consumer’s utility function; i.e., utility change can be measured in terms of income or expenditure change at the reference prices.

The \(i\)th Hicksian demand function, \(x_i(u, p)\) can be obtained by differentiating the expenditure function with respect to the \(i\)th consumer price, \(p_i\); i.e.:

\[
x_i(u, p) = \frac{\partial e(u, p)}{\partial p_i}, \quad i = 1, 2, 3, 4.
\]

The Hicksian demand functions defined by (5) have (unobservable) utility as an independent variable. An analytic expression for utility in period \(t\), \(u_t\), can be obtained by setting the expenditure function evaluated at period \(t\) utility, \(u_t\), and period \(t\) prices, \(p_t^t = (p_1^t, p_2^t, p_3^t, p_4^t)\), equal to period \(t\) expenditures on the 4 goods, \(Y_t\). Solving the resulting equation for \(u_t = G(Y_t^t, p_t)\), (the function \(g\) is the consumer’s indirect utility function) and substituting it into the equations (5), produces the following system of estimating equations:

\[
x_i^t = \frac{\partial g(Y_t^t, p_t^t)}{\partial p_i^t}, \quad i = 1, 2, 3, 4.
\]

To reduce heteroskedasticity, both sides of the \(i\)th equation in (6) are multiplied by \(p_i^t / Y_t^t\), which transforms (6) into a system of expenditure share equations.

Diewert and Lawrence (1994) used the consumer response model based on the transformed equations (6) and New Zealand data to estimate period \(t\) fitted demand for consumer goods using the non-linear regression program and the AUTO option in SHAZAM.\(^{24}\) The resulting parameter estimates are listed Table 4.2.

### Table 4.2: Parameter estimates for the consumer model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>(t) ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_1)</td>
<td>0.0175</td>
<td>0.24</td>
</tr>
<tr>
<td>(a_2)</td>
<td>0.3363</td>
<td>0.65</td>
</tr>
<tr>
<td>(a_3)</td>
<td>0.2305</td>
<td>8.85</td>
</tr>
<tr>
<td>(b_1)</td>
<td>0.0387</td>
<td>1.65</td>
</tr>
<tr>
<td>(b_2)</td>
<td>0.5119</td>
<td>3.12</td>
</tr>
<tr>
<td>(b_3)</td>
<td>-0.0074</td>
<td>-0.91</td>
</tr>
<tr>
<td>(c_1)</td>
<td>0.0641</td>
<td>1.36</td>
</tr>
</tbody>
</table>

---

\(^{23}\) The term money metric scaling is due to Samuelson (1974) but the concept may be found in Hicks (1946).

\(^{24}\) The same autocorrelation coefficient was estimated for each equation.
Since the period $t$ fitted demand for commodity $i$, $c_i^b$, is equal to the derivative of the estimated expenditure function with respect to the $i$th price evaluated at the period $t$ data, $u' = g(Y', p')$ and (i.e., we have ), the Hicksian price elasticity of demand for consumer good $i$ with respect to price $j$ can be defined as

$$h_{ij}^t = \frac{\partial \tilde{u}_i}{\partial p_j} / \tilde{x}_i \equiv \frac{\partial^2 e(\tilde{u}_i', p_j')}{\partial p_i \partial p_j} ; \quad i, j = 1, \ldots, 4$$

The sample means of the demand elasticities $h_{ij}^t$ are listed in Table 4.3.

### Table 4.3: Average compensated price elasticities of demand

<table>
<thead>
<tr>
<th>Change in quantity of:</th>
<th>With respect to price of:</th>
<th>Motor vehicles</th>
<th>General consumption</th>
<th>Housing</th>
<th>Leisure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor vehicles</td>
<td></td>
<td>-0.29</td>
<td>0.36</td>
<td>0.060</td>
<td>-0.14</td>
</tr>
<tr>
<td>General consumption</td>
<td></td>
<td>0.03</td>
<td>-0.41</td>
<td>0.005</td>
<td>0.38</td>
</tr>
<tr>
<td>Housing</td>
<td></td>
<td>0.05</td>
<td>0.05</td>
<td>-0.030</td>
<td>-0.07</td>
</tr>
<tr>
<td>Leisure</td>
<td></td>
<td>-0.02</td>
<td>0.82</td>
<td>-0.010</td>
<td>-0.79</td>
</tr>
</tbody>
</table>

From Table 6.3, it can be seen that the demand for housing (good 3) is quite inelastic: no price change significantly changes the demand for housing. The demand for motor vehicles (good 1) is also inelastic with respect to changes in the price of housing and the price of leisure (good 4), but motor vehicle demand is moderately substitutable with general consumption (good 2) since the average cross elasticity of demand for motor vehicles with respect to general consumption is 0.36. General consumption is quite substitutable with leisure since the two average cross elasticities are 0.38 and 0.82. The average price elasticity of demand for leisure, -0.79, is quite high in magnitude. This price elasticity of demand ranged between -0.64 in 1991 to -0.93 in 1976.

Hicksian real income elasticities of demand$^{25}$ can be defined as follows:

$$h_{iu}^t = \frac{\partial \tilde{u}_i}{\partial \tilde{u}_i} \frac{\partial^2 e(\tilde{u}_i', p_j')}{\partial p_i \partial p_j} ; \quad i = 1, \ldots, 4$$

$^{25}$ These elasticities are equal to ordinary income elasticities of demand.
Diewert and Lawrence’s sample average income elasticities were: $h_{1u} = 0.99$; $h_{2u} = 0.42$; $h_{3u} = 0.14$; and $h_{4u} = 2.40$ using New Zealand data. These results are somewhat surprising given that the income elasticity of demand for housing is generally assumed to exceed unity. However, over the first 16 observations in their sample, real income (or utility) changed very little. Consequently, it is likely that these estimates for income elasticities of demand are not very accurate. The average income elasticities over the last 4 observations of their data (when real incomes increased quite dramatically) were: $h_{1u} = 1.46$; $h_{2u} = 1.21$; $h_{3u} = 1.11$; and $h_{4u} = 5.13$. Thus, over the last four years in their sample, it appears that motor vehicles, housing and leisure all had income elasticities exceeding unity as expected.

4.3 Empirical results

Utilizing the Allais-Debreu excess burden concept discussed in Section 3.3 above, Diewert and Lawrence (1994) estimated marginal excess burdens. Since under Allais-Debreu excess burden concept utility is held constant, overall welfare is simply the value of government consumption of goods $G$ times the (constant) consumer price of general consumption $p_2$. Thus, welfare can be defined as a function of the exogenous tax and subsidy rates as follows:

(1) \[ W(t_1, t_2, t_4, t_5, s_2, s_3) \equiv P_2 G(t_1, t_2, t_4, t_5, s_2, s_3) . \]

The left hand side of the government budget constraint is essentially net government revenues and the right hand side is essentially government expenditures. Then, the net revenue function $R$ can be defined as a function of the exogenous tax and subsidy instruments as follows:

(2) \[ R(t_1, t_2, t_4, t_5, s_2, s_3) \equiv p_2(t_1, \ldots, s_3)G(t_1, \ldots, s_3) + p_5(t_1, \ldots, s_3)L_G + T(t_1, \ldots, s_3) \]

where \( G(t_1, \ldots, s_3), T(t_1, \ldots, s_3), p_1(t_1, \ldots, s_3), p_2(t_1, \ldots, s_3), P_3(t_1, \ldots, s_3) \), and \( p_5(t_1, \ldots, s_3) \) are the solution functions to the following system of simultaneous equations:

(3) \[ C_1 = Y_1 + S_1 ; \]

(4) \[ C_2 = Y_2 - I - G ; \]
The Allais-Debreu general equilibrium measure of the marginal excess burden associated with increasing the tax rate $t_i$, $MEB(t_i)$, is defined as in Section 3.3 as (minus) the rate of change in welfare defined by (1) divided by the rate of change in revenue defined by (2) with respect to $t_i$; i.e., for $i=1, 2, 4, 5, 6$:

\[ MEB(t_i) = \frac{-\partial W(t_1, \ldots, s_3) / \partial t_i}{\partial R(t_1, \ldots, s_3) / \partial t_i}. \]

Similar measures of marginal excess burden associated with decreasing the subsidy rate $s_j$ can be defined as follows for $j = 2, 3$:

\[ MEB(s_j) = \frac{-\partial W(t_1, \ldots, s_3) / \partial s_j}{\partial R(t_1, \ldots, s_3) / \partial s_j}. \]

Diewert and Lawrence (1994) evaluated the marginal excess burden measures defined by (9) and (10) using the elasticities and data generated by their models of producer and consumer behavior for New Zealand for the 20 years in their sample. The resulting marginal excess burdens are presented in Table 4.4.

**Table 4.4: Marginal excess burdens for New Zealand**

<table>
<thead>
<tr>
<th>Year</th>
<th>$MEB(t_1)$</th>
<th>$MEB(t_2)$</th>
<th>$MEB(t_4)$</th>
<th>$MEB(t_5)$</th>
<th>$MEB(s_2)$</th>
<th>$MEB(s_3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>-0.0341</td>
<td>0.049</td>
<td>0.019</td>
<td>0.053</td>
<td>0.037</td>
<td>0.038</td>
</tr>
<tr>
<td>1973</td>
<td>-0.0016</td>
<td>0.049</td>
<td>0.024</td>
<td>0.053</td>
<td>0.038</td>
<td>0.038</td>
</tr>
<tr>
<td>1974</td>
<td>-0.0027</td>
<td>0.053</td>
<td>0.027</td>
<td>0.057</td>
<td>0.043</td>
<td>0.038</td>
</tr>
<tr>
<td>1975</td>
<td>0.0071</td>
<td>0.071</td>
<td>0.029</td>
<td>0.075</td>
<td>0.056</td>
<td>0.065</td>
</tr>
<tr>
<td>1976</td>
<td>-0.0104</td>
<td>0.063</td>
<td>0.020</td>
<td>0.063</td>
<td>0.045</td>
<td>0.052</td>
</tr>
<tr>
<td>1977</td>
<td>-0.0057</td>
<td>0.070</td>
<td>0.026</td>
<td>0.073</td>
<td>0.054</td>
<td>0.056</td>
</tr>
<tr>
<td>1978</td>
<td>-0.0046</td>
<td>0.075</td>
<td>0.027</td>
<td>0.081</td>
<td>0.059</td>
<td>0.062</td>
</tr>
<tr>
<td>1979</td>
<td>-0.0026</td>
<td>0.065</td>
<td>0.023</td>
<td>0.067</td>
<td>0.048</td>
<td>0.047</td>
</tr>
<tr>
<td>1980</td>
<td>-0.0506</td>
<td>0.077</td>
<td>0.023</td>
<td>0.080</td>
<td>0.061</td>
<td>0.049</td>
</tr>
<tr>
<td>1981</td>
<td>-0.0349</td>
<td>0.075</td>
<td>0.022</td>
<td>0.079</td>
<td>0.058</td>
<td>0.047</td>
</tr>
<tr>
<td>1982</td>
<td>-0.0275</td>
<td>0.081</td>
<td>0.022</td>
<td>0.083</td>
<td>0.060</td>
<td>0.050</td>
</tr>
<tr>
<td>1983</td>
<td>-0.0510</td>
<td>0.077</td>
<td>0.020</td>
<td>0.083</td>
<td>0.058</td>
<td>0.041</td>
</tr>
</tbody>
</table>
According to Table 4.4, the important marginal excess burdens are $\text{MEB}_{t_2}$ and $\text{MEB}_{t_5}$, those burdens associated with increasing consumption and labor taxes. Both of these excess burdens are quite significant: an average of 8.3 percent for consumption taxation and an average of 9.5 percent for labor taxation. Because a general equilibrium framework is used here, these deadweight cost estimates apply year after year once a change in taxation has occurred. Consequently, if a government project is to be justified taking deadweight losses into account, it must provide a return each year which exceeds its direct cost (including a normal return) by at least the amount of the deadweight cost. This is equivalent to earning an ongoing real rate of return over and above the normal rate of return by at least the estimated percentage of deadweight costs. Thus, a government project financed by additional consumption (labor) taxation should earn a real rate of return 8.3 percent (9.5 percent) above the normal real rate of return in order to overcome the adverse effects of increased taxation. These are very large excess rates of return since in most countries the after tax real rate of return is about 3 percent.\footnote{We estimated the private sector’s average real rate of return to be 0.6% over the 20 years in our sample.}

However, the sample average excess burdens do not tell the whole story. Examination of Table 4.4 show that as tax rates in the New Zealand economy increased, marginal excess burdens have also tended to increase. Thus, the marginal excess burden associated with increased consumption (labor) taxation grew from 4.9 percent (5.3 percent) in 1972 to 13.7 percent (18.3 percent) in 1991. These trends are shown in Figures 4.2 and 4.4.

It should be noted that the increase in the deadweight cost of labor taxation is not accounted for by increases in labor tax rates, from 20 percent to 32 percent over the same period. Although deadweight costs tend to increase more rapidly than the increase in the tax rate (all else equal), much of the increase in the size of the deadweight cost of labor taxation is probably explained by the increased flexibility and responsiveness of the New Zealand economy in recent years and increasing international capital mobility. This can
be seen from Figure 4.2 where the rate of increase in the labor tax rate eased off after 1983 while the deadweight cost of labor taxation increased rapidly after 1984.

Figure 4.2: **Labor Tax Rates and Deadweight Costs**

![Graph showing Labor Tax Rates and Deadweight Costs]

Table 4.3 also shows that the marginal excess burden associated with increasing the tax rate on international trade averaged 2.6 percent during the sample period, showing only a gradual upward trend. The general tendency for excess burdens to increase markedly over time was probably offset in this case by the reductions in trade taxes that took place in New Zealand during the 1970s and 1980s.

Table 4.3 also shows that the average marginal excess burdens associated with financing increased government expenditures by reducing the subsidy rate to domestic output producers $s_2$ and to exporters $s_3$ were 7.0 percent and 5.2 percent, respectively. This means that increasing $s_2$ or $s_3$ would produce marginal benefits. The reason for this result is that increasing $s_2$ is approximately equivalent to reducing the general output tax rate $t_2$ and increasing $s_3$ is approximately equivalent to reducing the tariff rate $t_4$. Hence, these
increases in subsidy rates tend to move the economy towards a more optimal tax structure and hence reduce excess burdens. Looking at trends in the marginal excess burdens associated with decreasing subsidies, we see that $MEB(s_2)$ trends upward from 3.7 percent in 1972 to 14.1 percent in 1991 while $MEB(s_3)$ trends upward from 3.8 percent to 6.6 percent. The more rapid growth in $MEB(s_2)$ reflects the more rapid growth in $t_2$ compared to $t_4$ over the 20 years in New Zealand.

Finally, Table 4.3 tells us that the marginal excess burden of financing increased government expenditures by increasing the tax rate $t_1$ on new motor vehicles is actually a marginal excess benefit which averaged 2.53 percent over the sample period. This means that, on average, a government project financed by increased motor vehicle taxation could earn a real rate of return which was 2.53 percent below the normal real rate of return and consumer overall welfare would remain unchanged. This result is due to the fact that motor vehicles are complementary to many goods, both in consumption and production.

According to Table 4.3, the marginal excess burden associated with increasing the profits tax was zero in each period and hence not listed in Table 4.3. As noted in section 3, this zero excess burden result is entirely due to the simplifying assumption that investment was exogenously determined and was therefore unaffected by capital taxation. We know that this is not the case.27

The results of this study are well within the range of deadweight loss estimates of previous studies for other countries. The model uses a rigorous general equilibrium framework and its key parameters are based on econometric estimates derived from flexible functional forms using the latest techniques rather than the guesstimates and restrictive assumptions of earlier studies. The results are all plausible. For instance, the finding that increasing subsidy rates on both general production and exports actually improves welfare by acting to partially offset the adverse effects of taxes on general production and imports further highlights the costs of taxation and the distortions in the allocation of resources they cause.

Nevertheless, it is also subject to a number of limitations: (i) the model is static; investment and capital accumulation decisions have not been modeled; (ii) in modeling the labor supply decision, it was assumed that all unemployment was voluntary and the discrete aspect of being in the labor force was ignored; (iii) only one class of household was

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considered and hence may be subject to aggregation over consumers bias; (iv) the model had only 7 goods in it and hence may be subject to some aggregation over commodities bias; (v) average and marginal tax rates were assumed to be the same; (vi) the elasticity estimates were biased downwards\(^{28}\) and hence it is probable that the excess burden measures were also biased downwards.

The priority for future applied work should be to evaluate the effects of these simplifications and to extend marginal excess burden models to make them intertemporal and to include explicit modeling of the capital accumulation process.

Another priority is to extend excess burden models like this to other instruments of government, especially government regulation of business. Fitting government regulation of business into this framework will be an especially difficult task, because in many instances we currently lack good measures of the direct and enforcement costs of regulation. That lack must be rectified before we can begin to extend marginal excess burden models to regulation.

**FINAL OBSERVATIONS**

Not so long ago public finance economists tended to be preoccupied with two questions. The first is the tax incidence question -- who bears the cost of taxation? The second is the equity question -- which itself has two aspects, vertical equity -- are burdens matched to ability to pay, and horizontal equity -- do people with the same ability to pay bear the same tax burdens? Nowadays most public finance economists understand that these questions are really less important from a policy standpoint than the efficiency question: are government services worth more than they cost? Or, perhaps, how does one fix things, so that government services are worth more than they cost? Of course, it is usually necessary to first answer the incidence question in order to answer the efficiency question precisely.

As the preceding chapters in this volume explain there are practical analytical reasons for this shift from a concern with redistribution and and uniform taxation to one that plays down distributional concerns. Horizontal equity, for example, turns out to be a lot less

\(^{28}\) This downward bias is due to the fact that we have frozen the allocation of capital during each year. The Le Chatelier Principle of Samuelson (1947; 36-38) and Hicks (1946; 206) suggests that long run elasticities will be bigger than short run elasticities; see also Diewert (1974; 146-150) (1985b; 224). Also Diewert (1985b; 237) shows that excess burdens in a dynamic model will generally exceed the discounted present value of corresponding static excess burdens; i.e., growth can only augment the effect of static distortions (at least to the second order).
important than previously thought, in part because differences in taxation, especially those involving income from capital, are reflected in asset prices and different before-tax returns. And, while vertical equity may be important, is difficult to implement because of timing issues and the practical barriers to taxing income from owner-enjoyed assets such as housing or leisure.

To these factors this chapter adds the information that vertical equity is also exceedingly costly when pursued via progressive taxation. That is to say, the disincentive and distortionary effects of high marginal tax rates are likely to overwhelm any social benefits from their redistributive effects. This chapter also shows that the source of financing can substantially increase the cost of governmental services and, therefore, their attractiveness. Both those who want greater services from government and those who want to minimize the burdens that government imposes upon the citizenry should have a substantial interest finding ways of reducing the deadweight burdens of taxation.

Holding spending constant, this implies a preference for taxes with low rates and broad bases -- a portfolio of tax types if you will rather than reliance on a few measures. From this perspective, standard American practice, for example, where local governments and schools finance themselves with property taxes and user fees, states with broad-based retail sales taxes, and the federal government relies on an income tax, has much to recommend it, although we would tinker with things at the margin (i.e., the property tax base should be broadened in most jurisdictions to comprehend all real property). However, precision tax engineering requires better empirical understanding of the deadweight burdens of taxation (and regulation) than anyone has right now. In the mean time, the surest way to reduce the costs of taxation is to reduce government spending.
REFERENCES


International Monetary Fund (various years), Government Finance Statistics Yearbook, New York.


