Risk and Return: Portfolio Theory and Asset Pricing Models

Portfolio Theory

Capital Asset Pricing Model (CAPM)

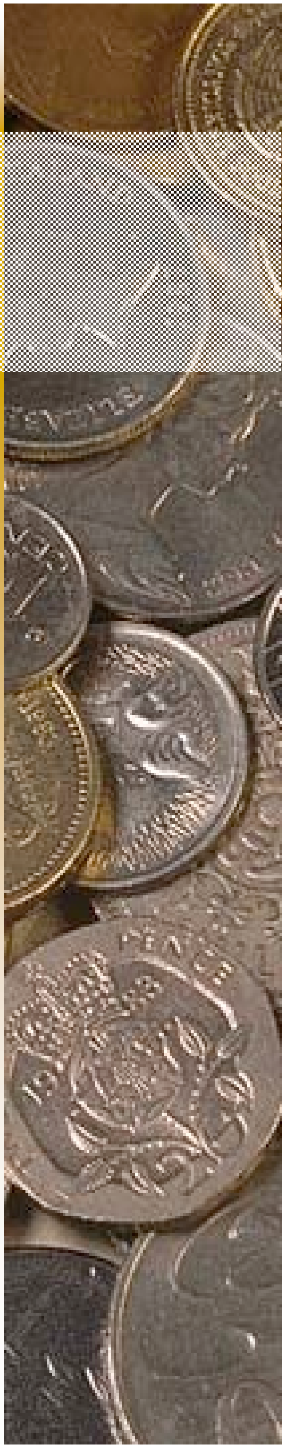
Efficient frontier
Capital Market Line (CML)
Security Market Line (SML)
Beta calculation

Arbitrage pricing theory

Fama-French 3-factor model

Portfolio Theory

- Suppose Asset A has an expected return of 10 percent and a standard deviation of 20 percent. Asset B has an expected return of 16 percent and a standard deviation of 40 percent. If the correlation between A and B is 0.6, what are the expected return and standard deviation for a portfolio comprised of 30 percent Asset A and 70 percent Asset B?

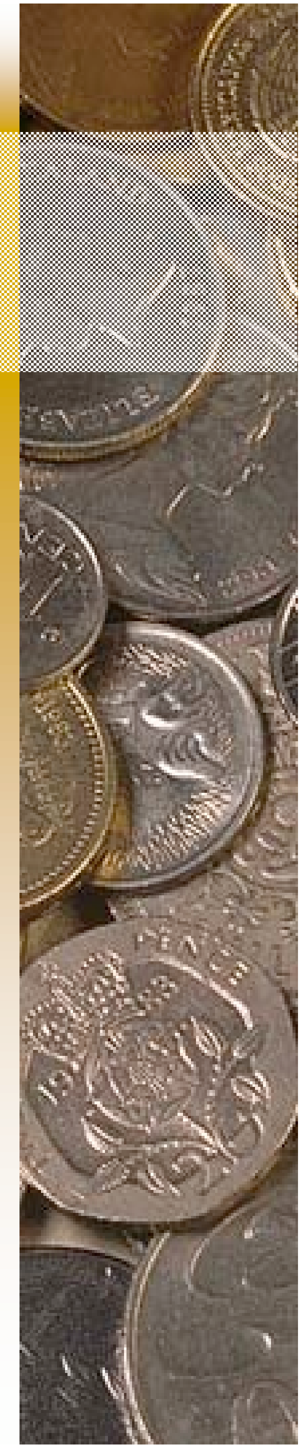


Portfolio Expected Return

$$\hat{r}_P = w_A \hat{r}_A + (1 - w_A)\hat{r}_B$$

$$= 0.3(0.1) + 0.7(0.16)$$

$$= 0.142 = 14.2\%.$$

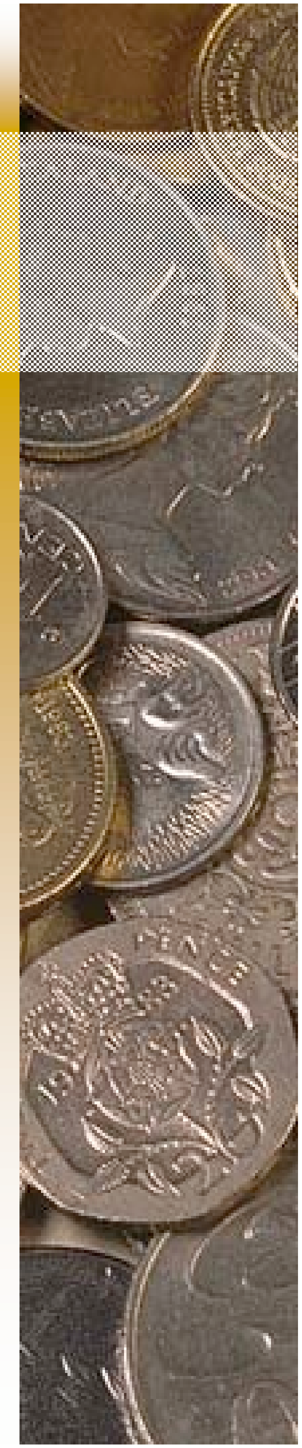


Portfolio Standard Deviation

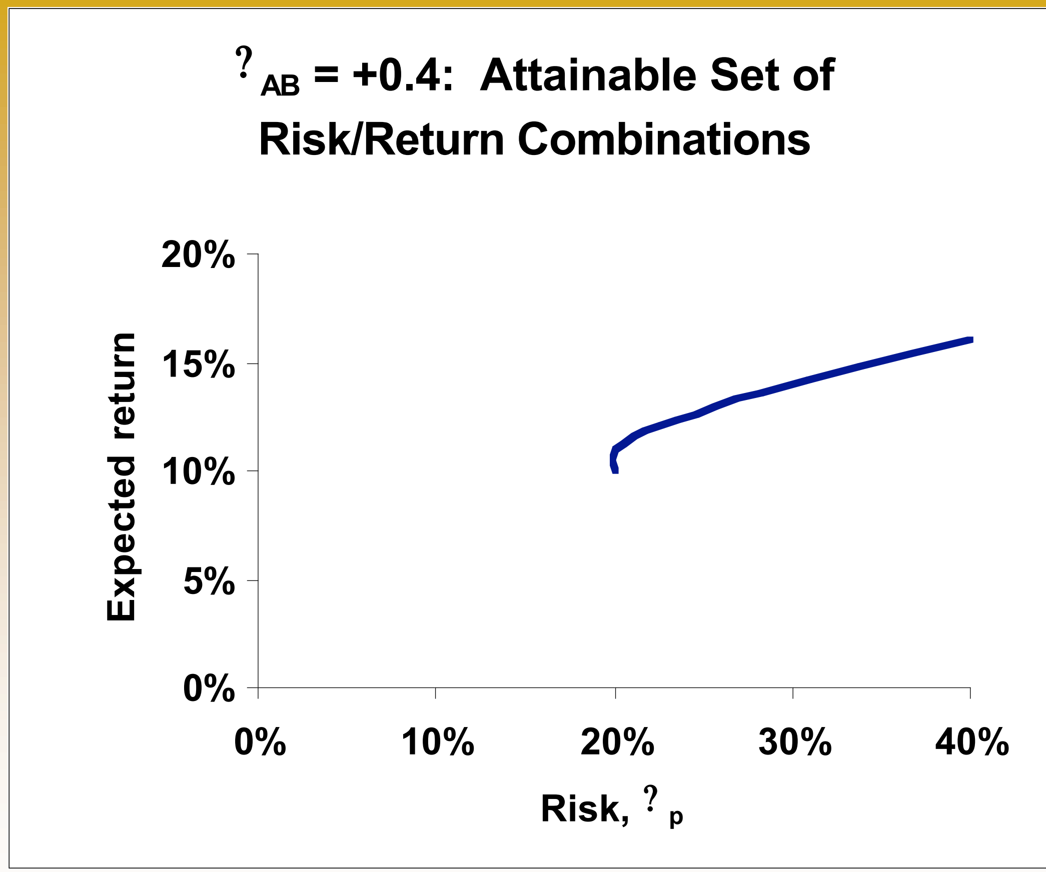
$$\sigma_p = \sqrt{W_A^2 \sigma_A^2 + (1 - W_A)^2 \sigma_B^2 + 2W_A(1 - W_A)\rho_{AB}\sigma_A\sigma_B}$$

$$= \sqrt{0.3^2(0.2^2) + 0.7^2(0.4^2) + 2(0.3)(0.7)(0.4)(0.2)(0.4)}$$

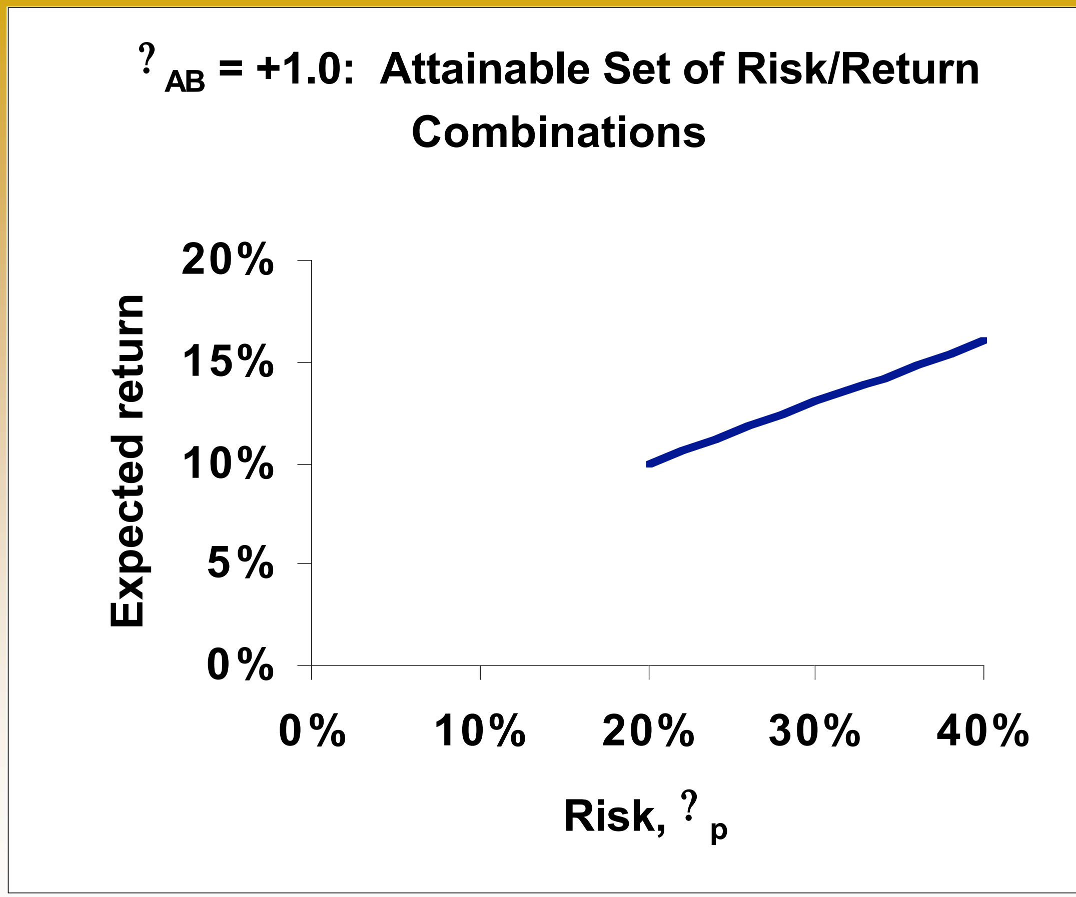
$$= 0.309$$



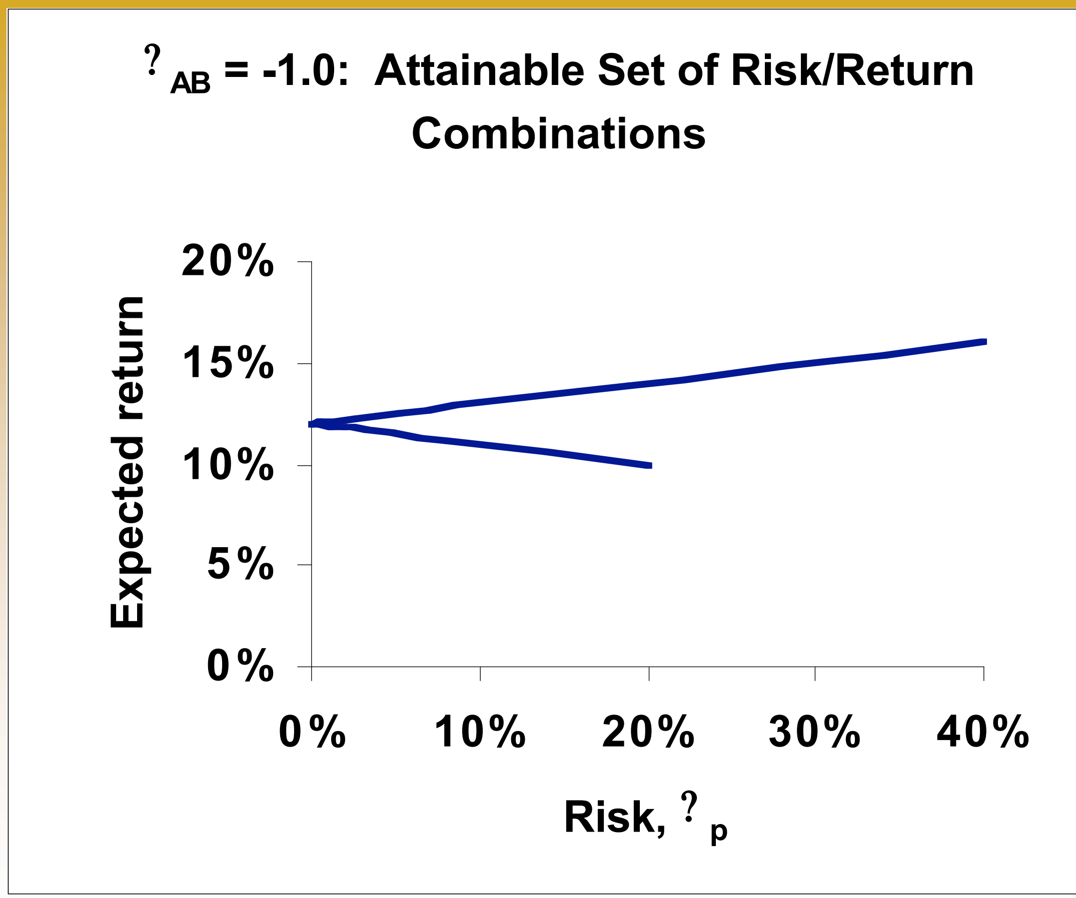
Attainable Portfolios: $\square_{AB} = 0.4$

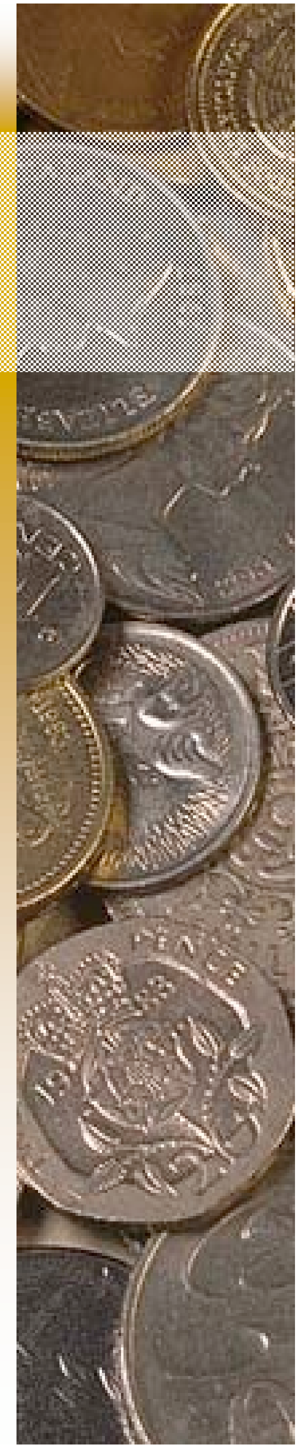


Attainable Portfolios: $\rho_{AB} = +1$

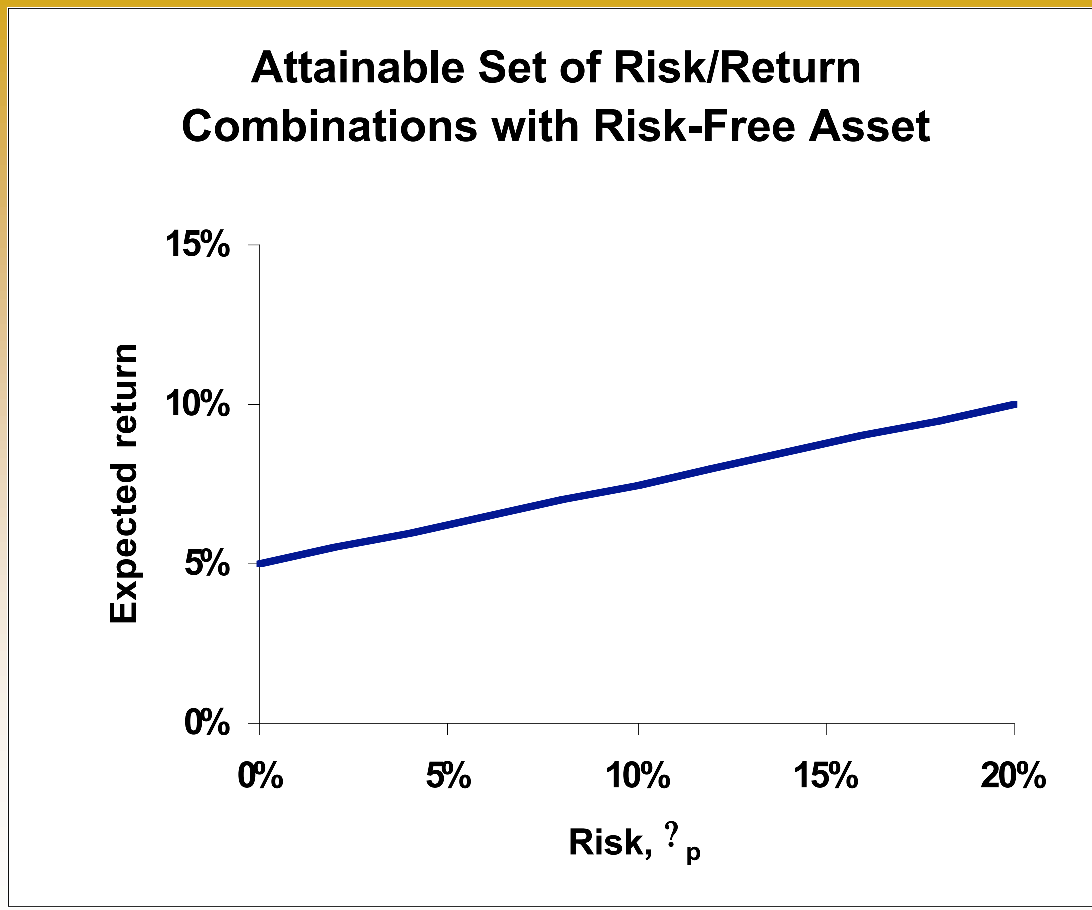


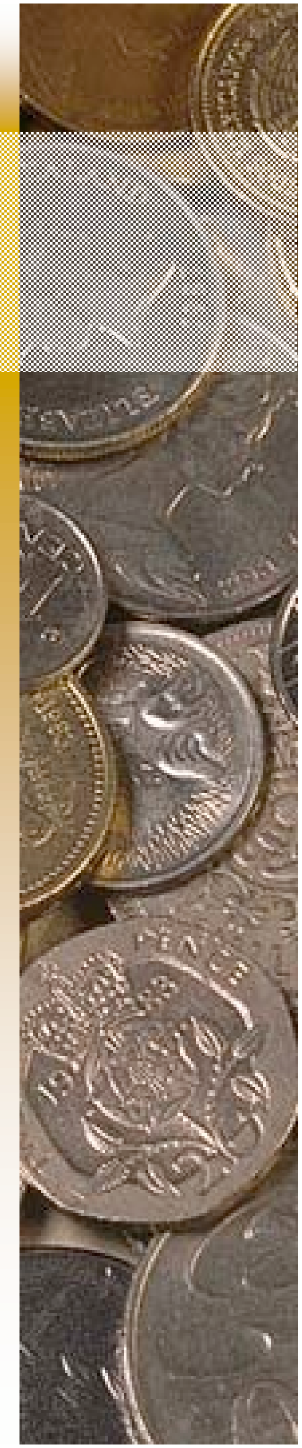
Attainable Portfolios: $\rho_{AB} = -1$

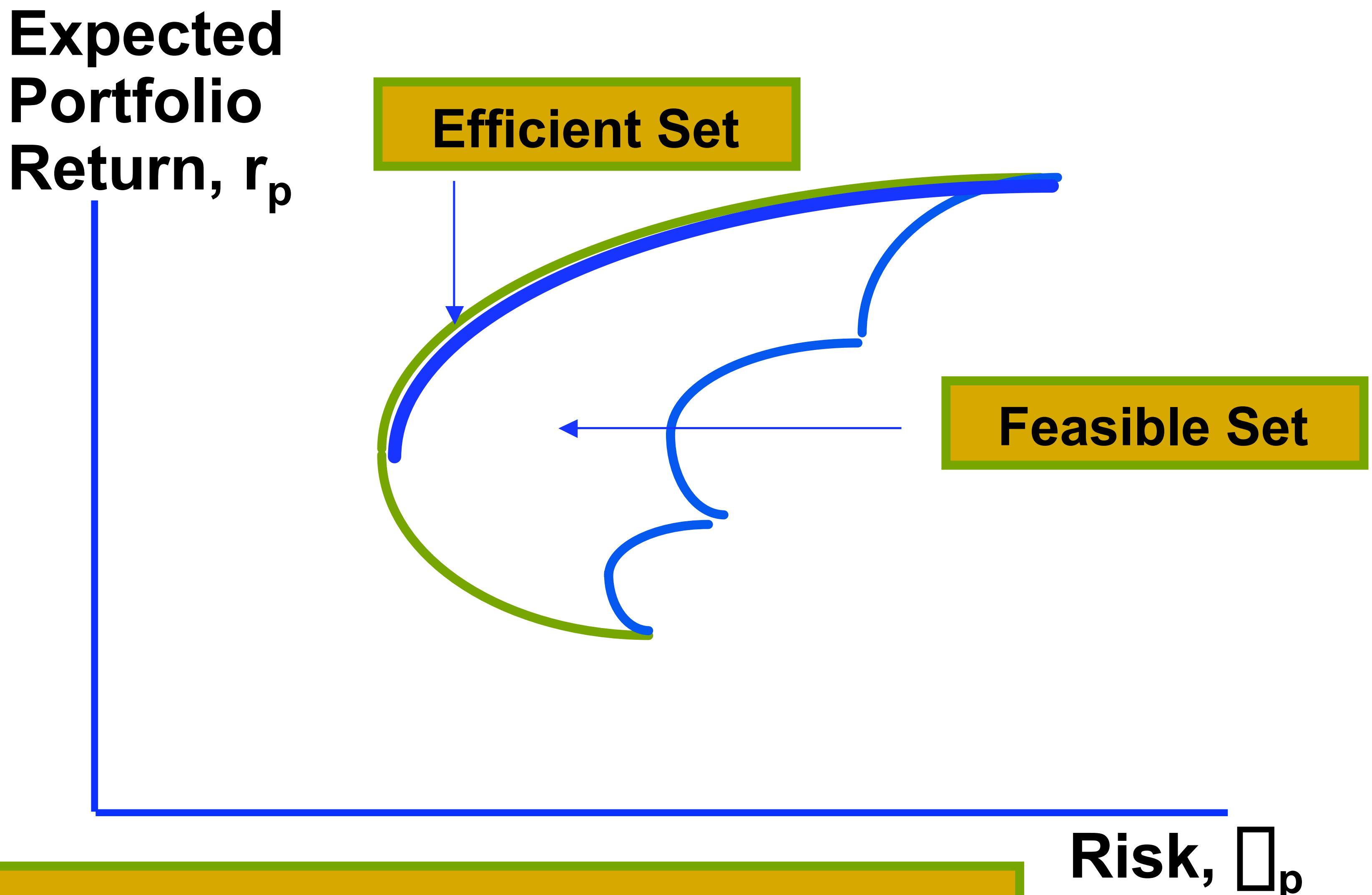




Attainable Portfolios with Risk-Free Asset (Expected risk-free return = 5%)

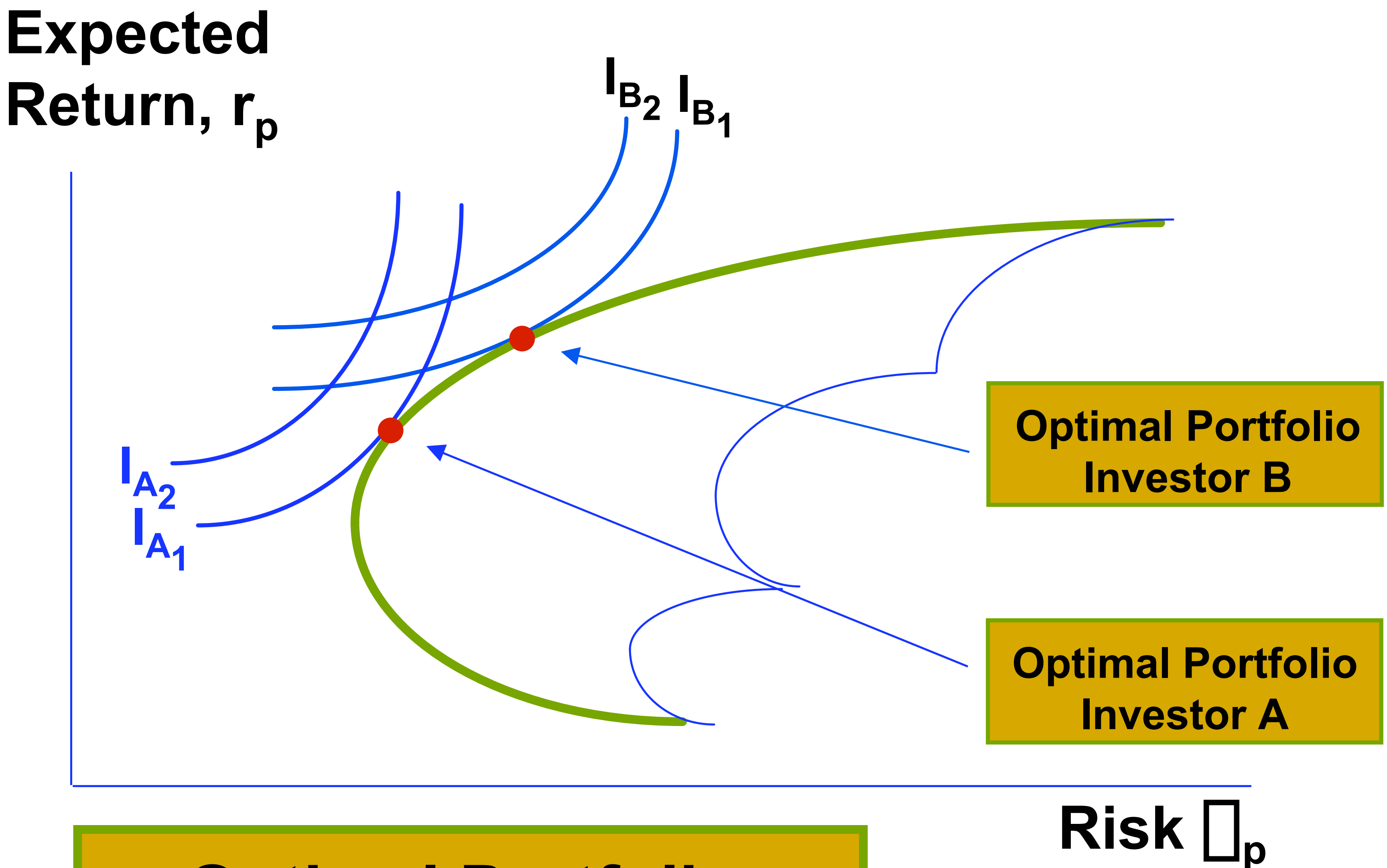






Feasible and Efficient Portfolios

- The feasible set of portfolios represents all portfolios that can be constructed from a given set of stocks.
- An efficient portfolio is one that offers:
  - the most return for a given amount of risk, or
  - the least risk for a give amount of return.
- The collection of efficient portfolios is called the efficient set or efficient frontier.

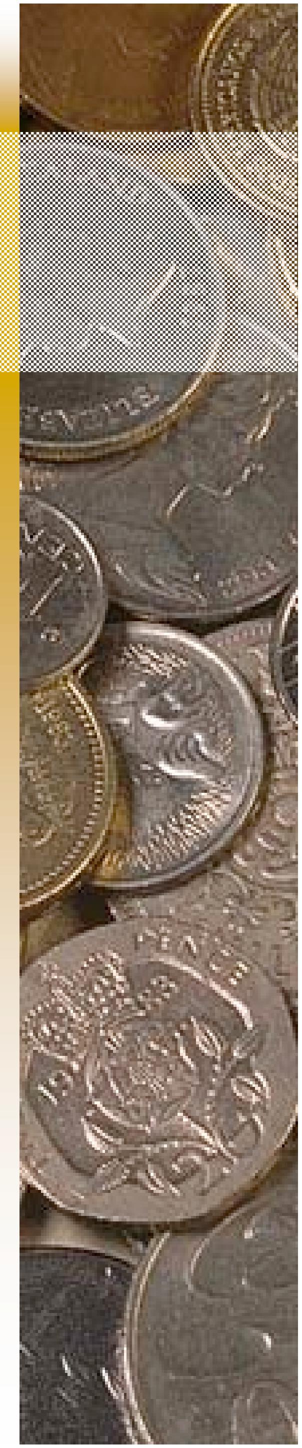


Optimal Portfolios

- Indifference curves reflect an investor's attitude toward risk as reflected in his or her risk/return tradeoff function. They differ among investors because of differences in risk aversion.
- An investor's optimal portfolio is defined by the tangency point between the efficient set and the investor's indifference curve.

What is the CAPM?

- The CAPM is an equilibrium model that specifies the relationship between risk and required rate of return for assets held in well-diversified portfolios.
- It is based on the premise that only one factor affects risk.
- What is that factor?



What are the assumptions of the CAPM?

- Investors all think in terms of a single holding period.
- All investors have identical expectations.
- Investors can borrow or lend unlimited amounts at the risk-free rate.

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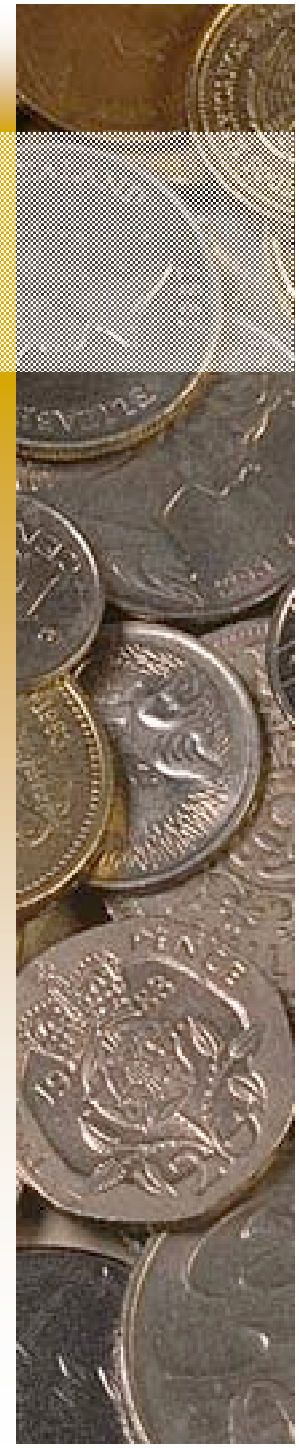
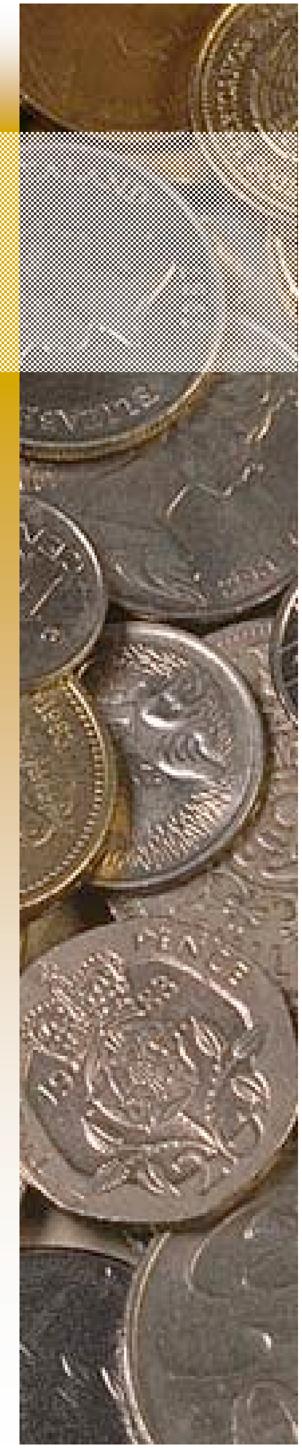
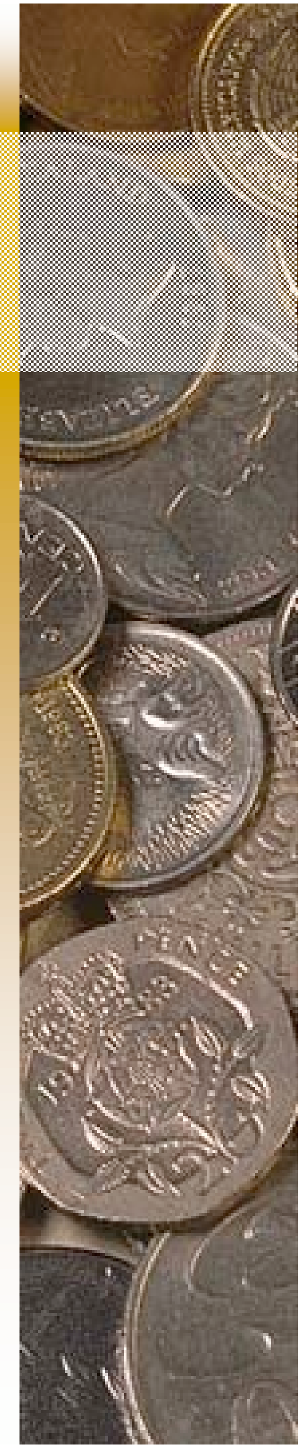


What are the assumptions of the CAPM?



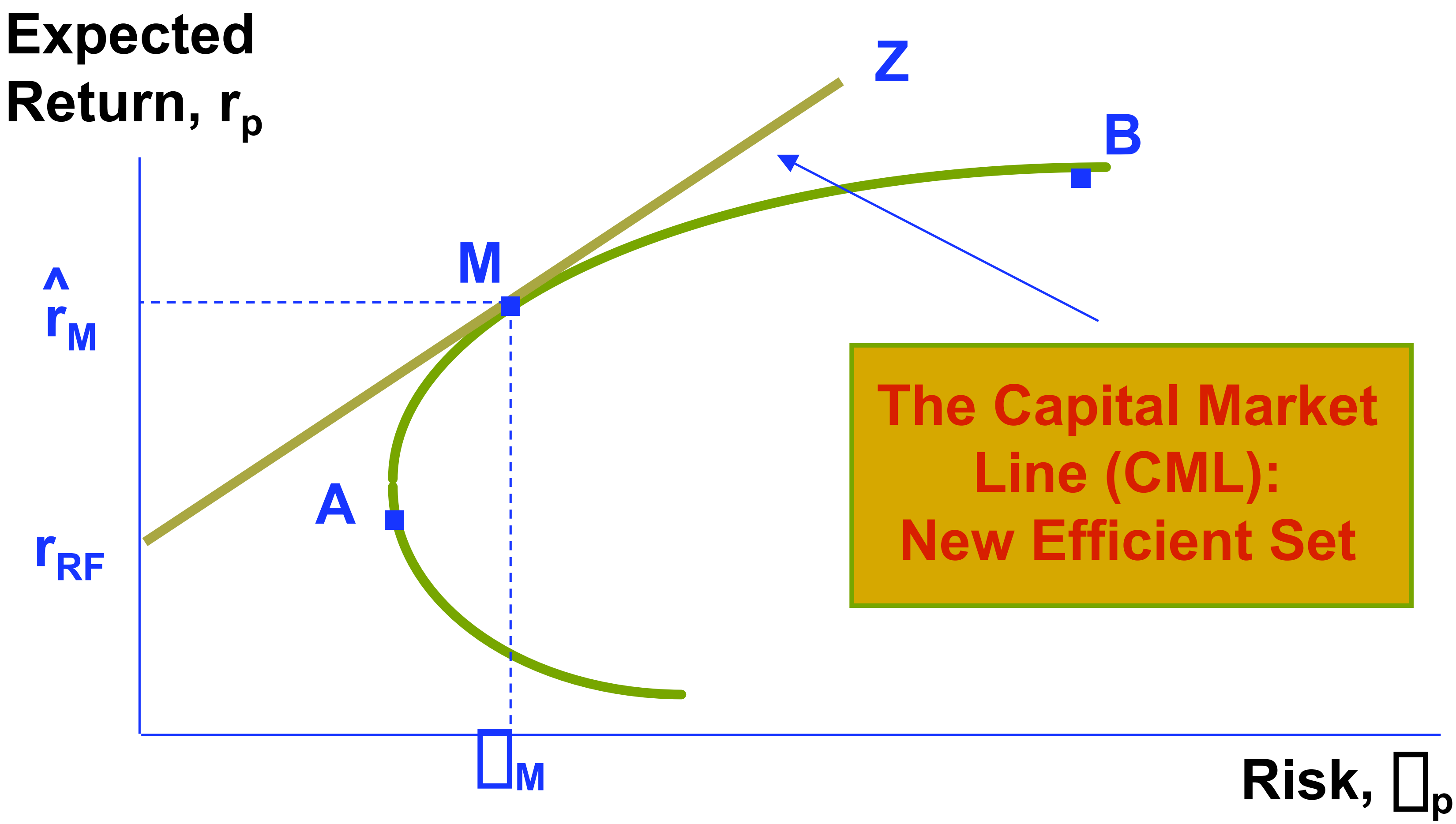
- All assets are perfectly divisible.
- There are no taxes and no transactions costs.
- All investors are price takers, that is, investors' buying and selling won't influence stock prices.
- Quantities of all assets are given and fixed.

What impact does r_{RF} have on the efficient frontier?

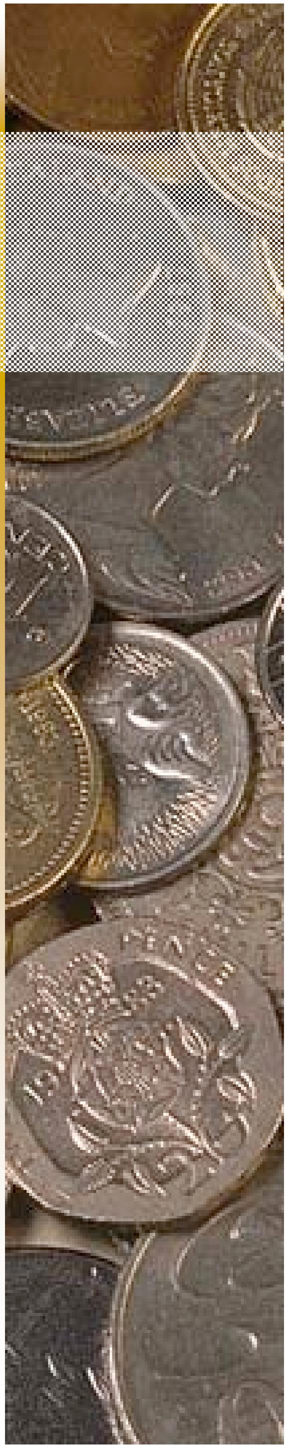


- When a risk-free asset is added to the feasible set, investors can create portfolios that combine this asset with a portfolio of risky assets.
- The straight line connecting r_{RF} with M, the tangency point between the line and the old efficient set, becomes the new efficient frontier.

Efficient Set with a Risk-Free Asset



What is the Capital Market Line?



- The Capital Market Line (CML) is all linear combinations of the risk-free asset and Portfolio M.
- Portfolios below the CML are inferior.
  - The CML defines the new efficient set.
  - All investors will choose a portfolio on the CML.

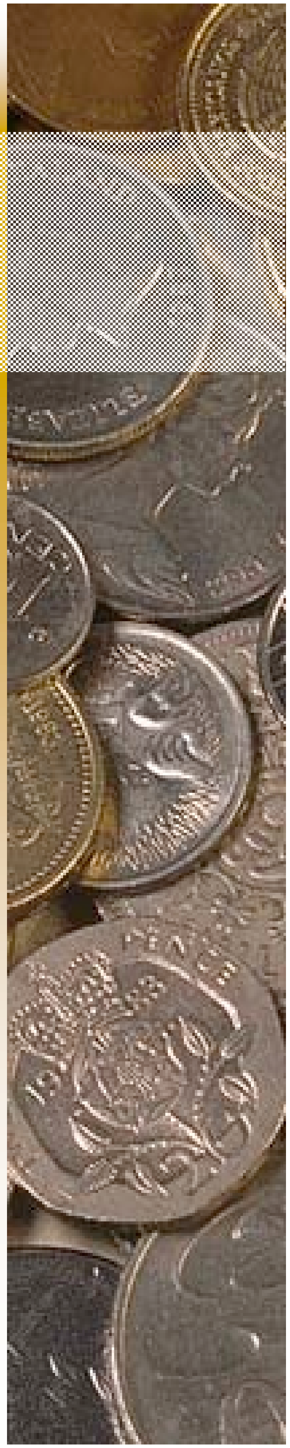
The CML Equation

$$\hat{r}_p = r_{RF} + \left[\frac{\hat{r}_M - r_{RF}}{\sigma_M} \right] \sigma_p.$$

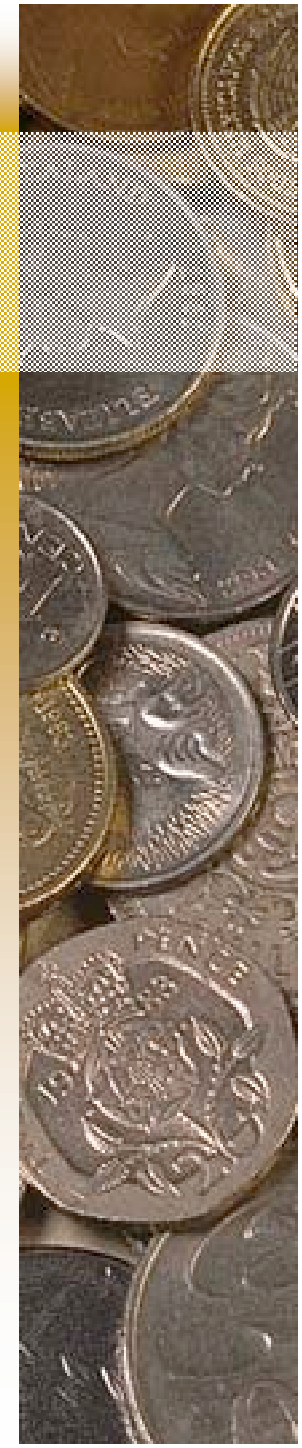
Intercept

Slope

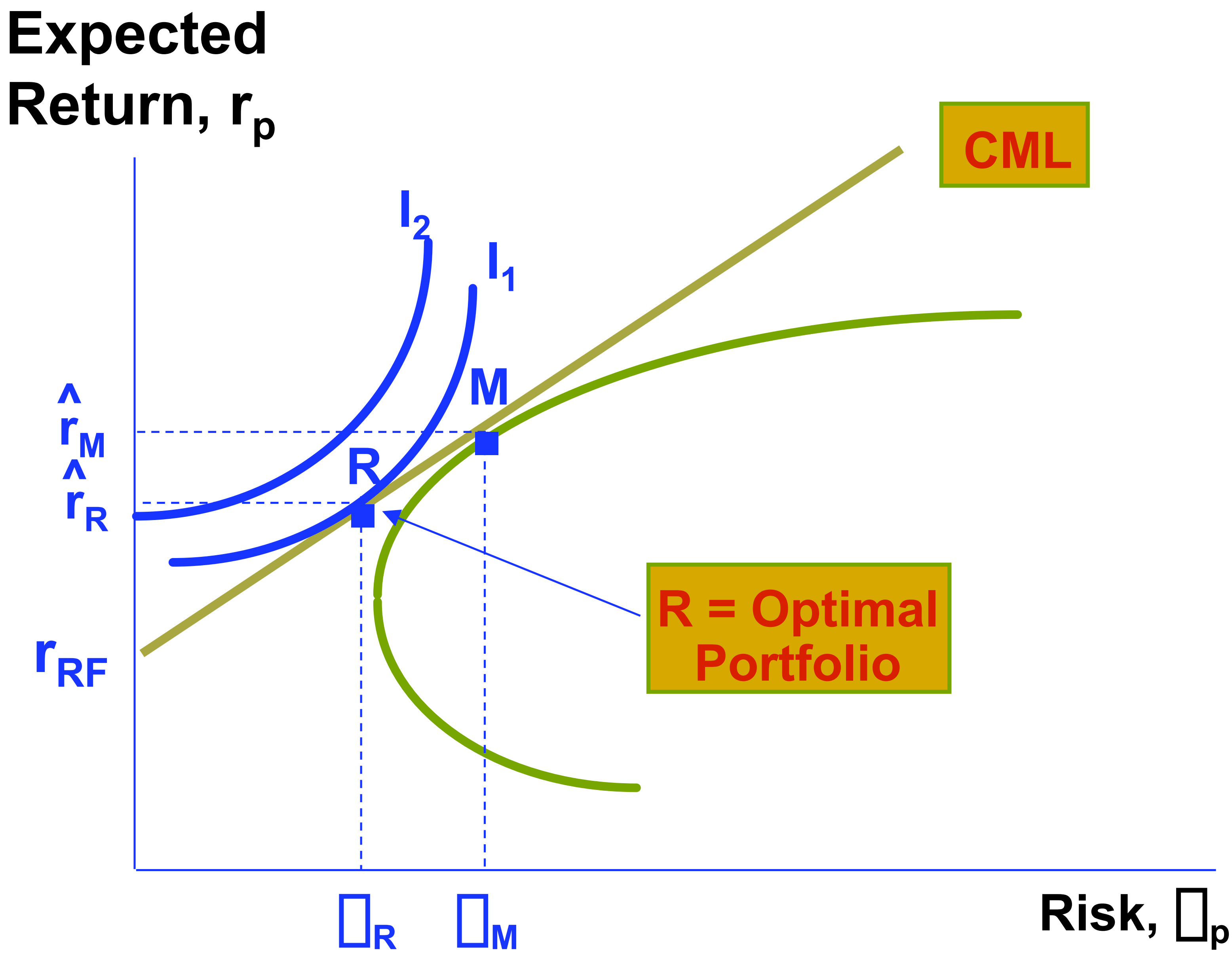
Risk measure



What does the CML tell us?



- The expected rate of return on any efficient portfolio is equal to the risk-free rate plus a risk premium.
- The optimal portfolio for any investor is the point of tangency between the CML and the investor's indifference curves.



Expected
Return, rp
CML
I2
I1
M
R
^rM
^rR
rRF
R = Optimal
Portfolio
□R
□M
Risk, □p

What is the Security Market Line (SML)?

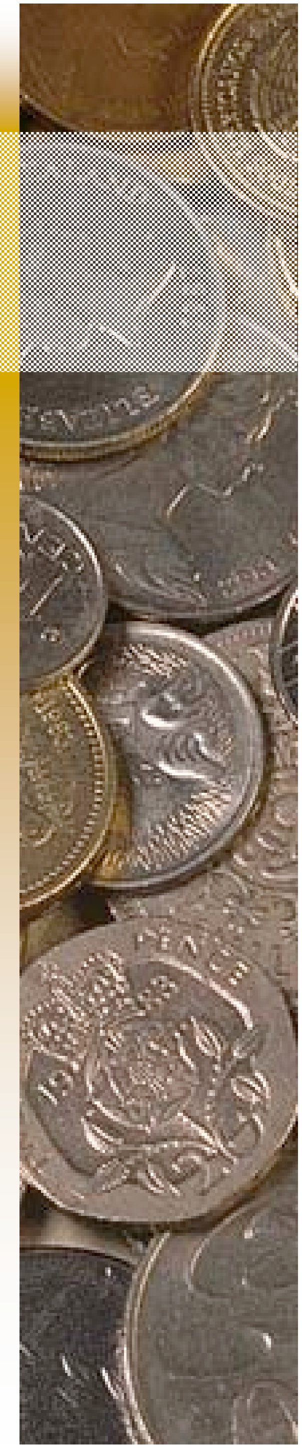
- The CML gives the risk/return relationship for efficient portfolios.
- The Security Market Line (SML), also part of the CAPM, gives the risk/return relationship for individual stocks.

The SML Equation

- The measure of risk used in the SML is the **beta coefficient** of company i, b_i.

- The SML equation:

$$r_i = r_{RF} + (RP_M)\, b_i$$



How are betas calculated?

- Run a regression line of past returns on Stock i versus returns on the market.
- The regression line is called the characteristic line.
- The slope coefficient of the characteristic line is defined as the beta coefficient.

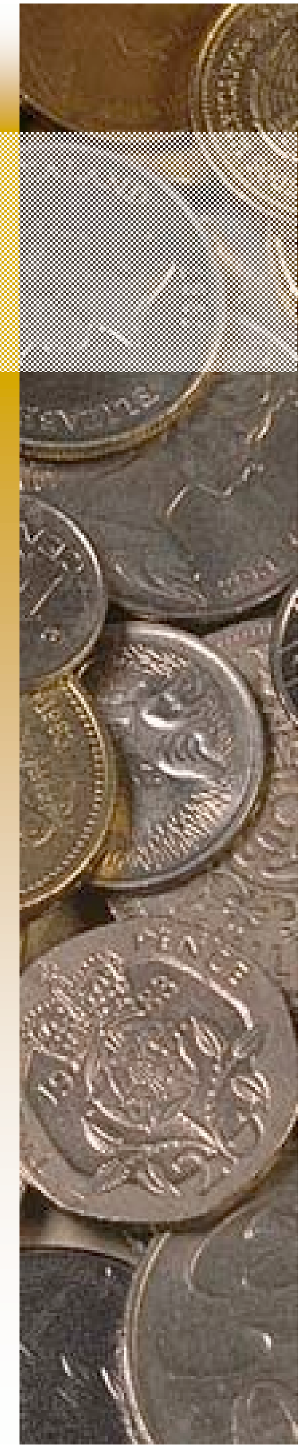
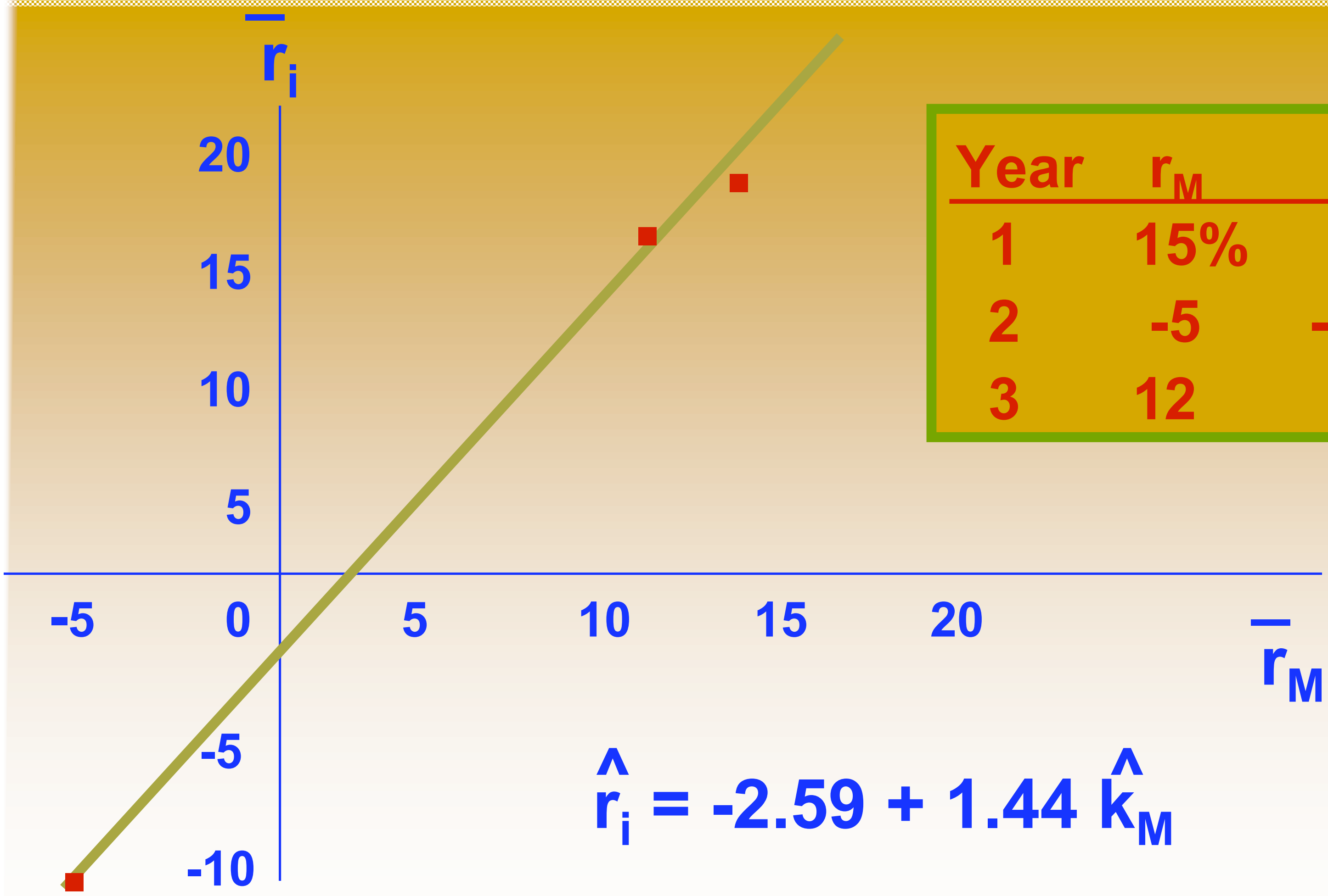
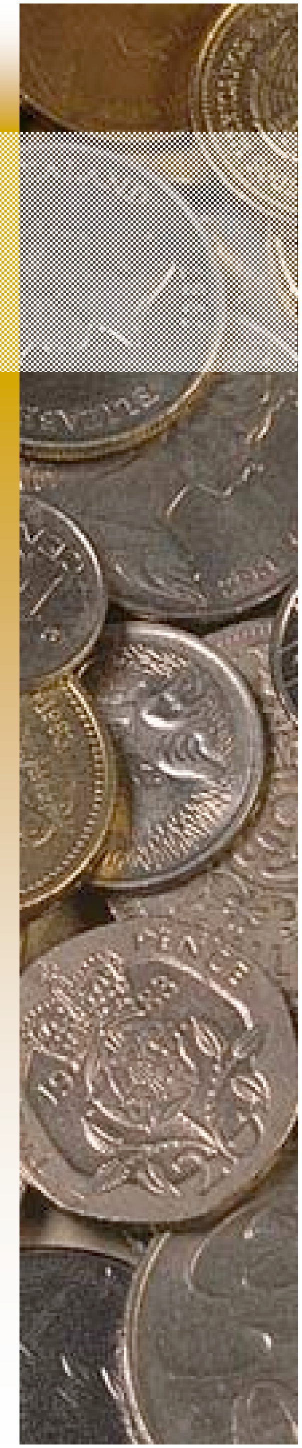


Illustration of beta calculation



Year	r_M	r_i
1	15%	18%
2	-5	-10
3	12	16

$$\hat{r}_i = -2.59 + 1.44\,\hat{k}_M$$



Method of Calculation

- Analysts use a computer with statistical or spreadsheet software to perform the regression.
  - At least 3 year's of monthly returns or 1 year's of weekly returns are used.
  - Many analysts use 5 years of monthly returns.

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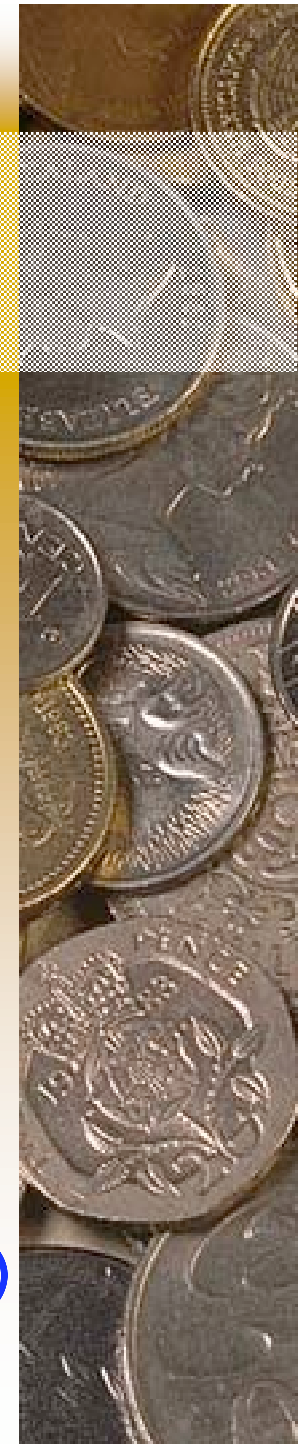


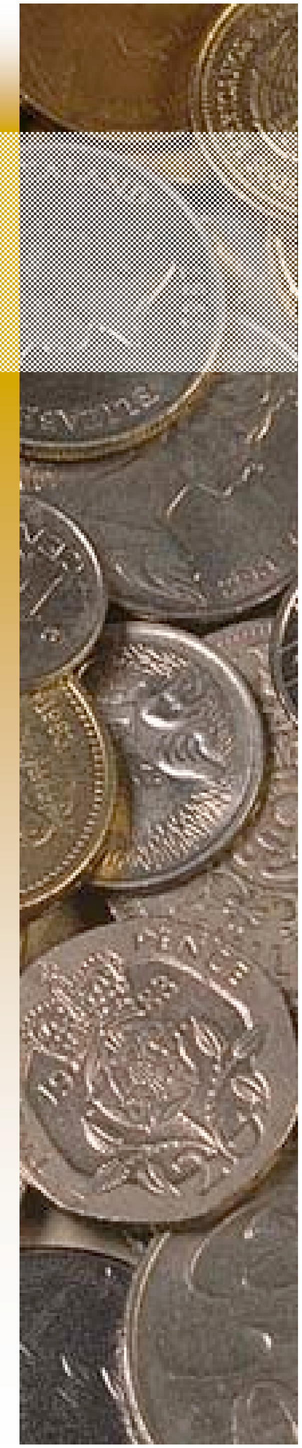
- If beta = 1.0, stock is average risk.
- If beta > 1.0, stock is riskier than average.
- If beta < 1.0, stock is less risky than average.
- Most stocks have betas in the range of 0.5 to 1.5.

Interpreting Regression Results

- The R^2 measures the percent of a stock's variance that is explained by the market. The typical R^2 is:
  - 0.3 for an individual stock
  - over 0.9 for a well diversified portfolio

Interpreting Regression Results (Continued)

- The 95% confidence interval shows the range in which we are 95% sure that the true value of beta lies. The typical range is:
  - from about 0.5 to 1.5 for an individual stock
  - from about .92 to 1.08 for a well diversified portfolio



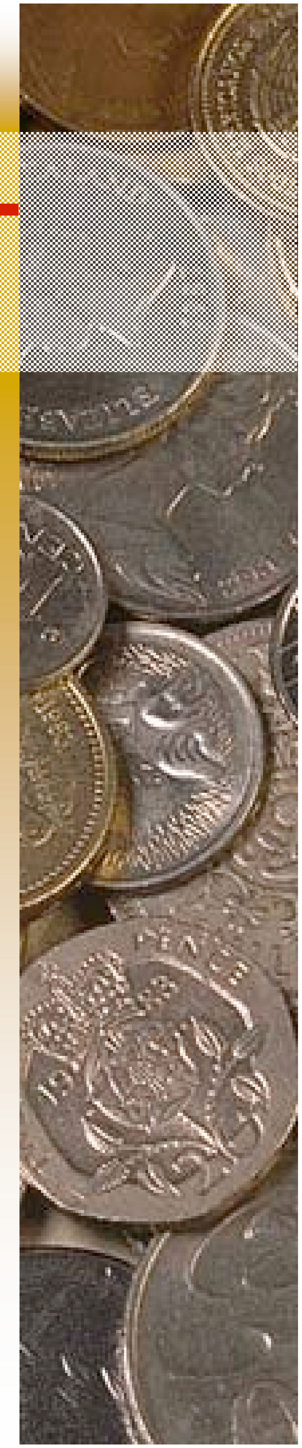
What is the relationship between stand-alone, market, and diversifiable risk.

$$\sigma_j^2 = b_j^2 \sigma_M^2 + \sigma_{e_j}^2.$$

σ_j^2 = variance
= stand-alone risk of Stock j.

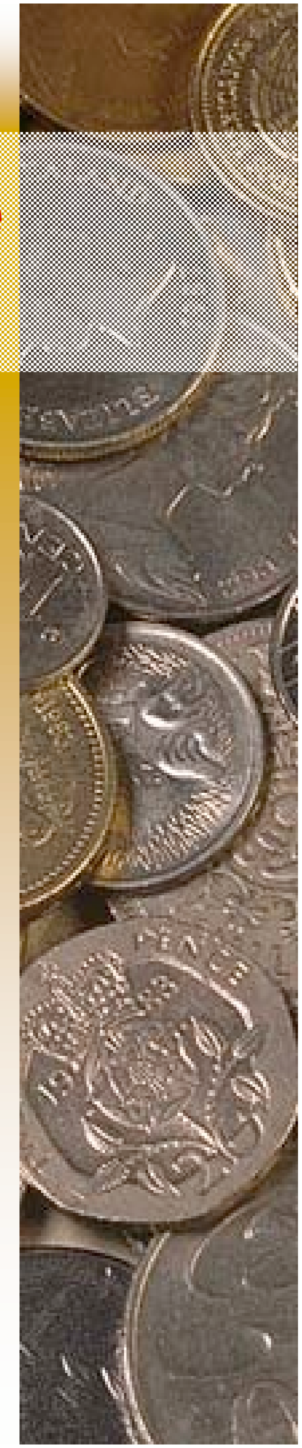
$b_j^2 \sigma_M^2$ = market risk of Stock j.

$\sigma_{e_j}^2$ = variance of error term
= diversifiable risk of Stock j.



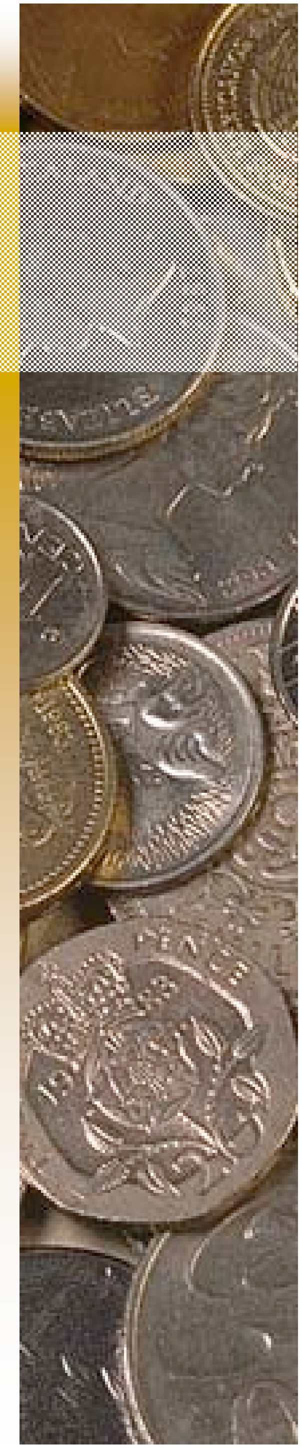
What are two potential tests that can be conducted to verify the CAPM?

- Beta stability tests
- Tests based on the slope of the SML



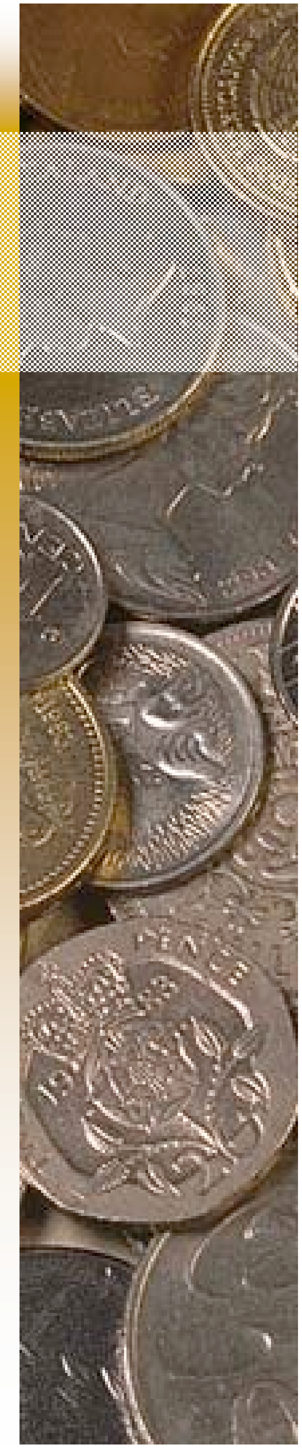
Tests of the SML indicate:

- A more-or-less linear relationship between realized returns and market risk.
- Slope is less than predicted.
- Irrelevance of diversifiable risk specified in the CAPM model can be questioned.
- Betas of individual securities are not good estimators of future risk.
- Betas of portfolios of 10 or more randomly selected stocks are reasonably stable.
- Past portfolio betas are good estimates of future portfolio volatility.



Are there problems with the CAPM tests?

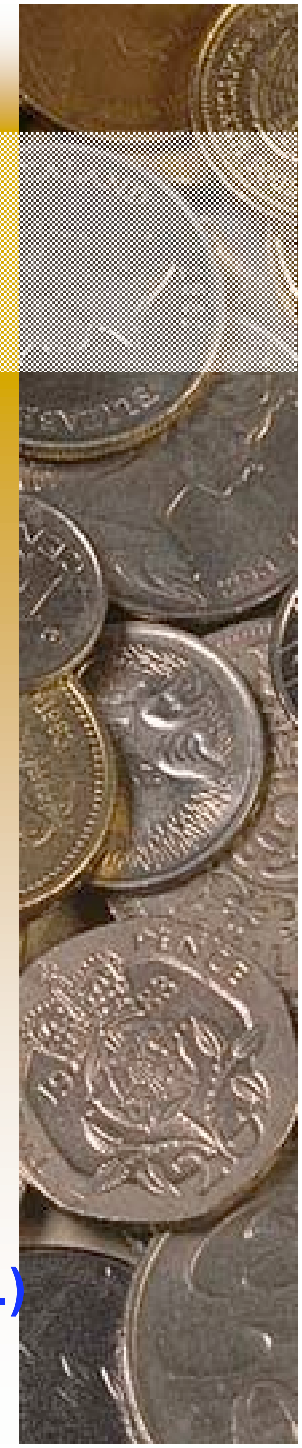
- Yes.
  - Richard Roll questioned whether it was even conceptually possible to test the CAPM.
  - Roll showed that it is virtually impossible to prove investors behave in accordance with CAPM theory.



What are our conclusions regarding the CAPM?

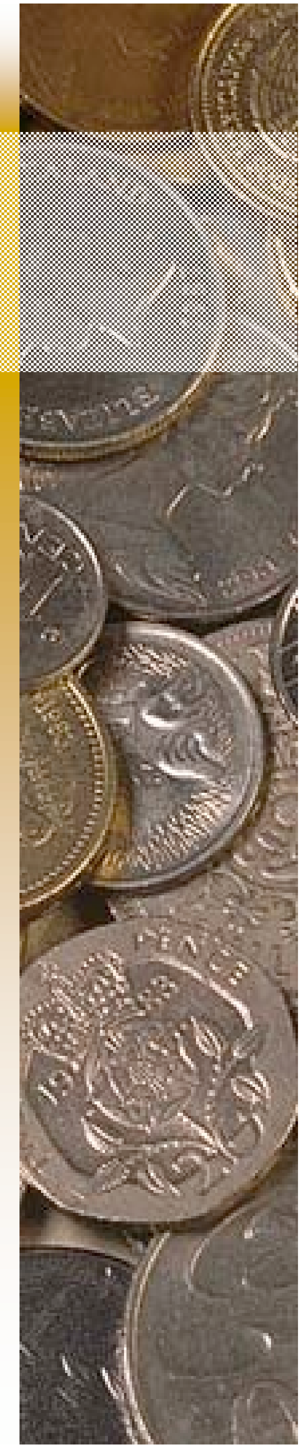
- It is impossible to verify.
- Recent studies have questioned its validity.
- Investors seem to be concerned with both market risk and stand-alone risk. Therefore, the SML may not produce a correct estimate of r_i.

(More...)



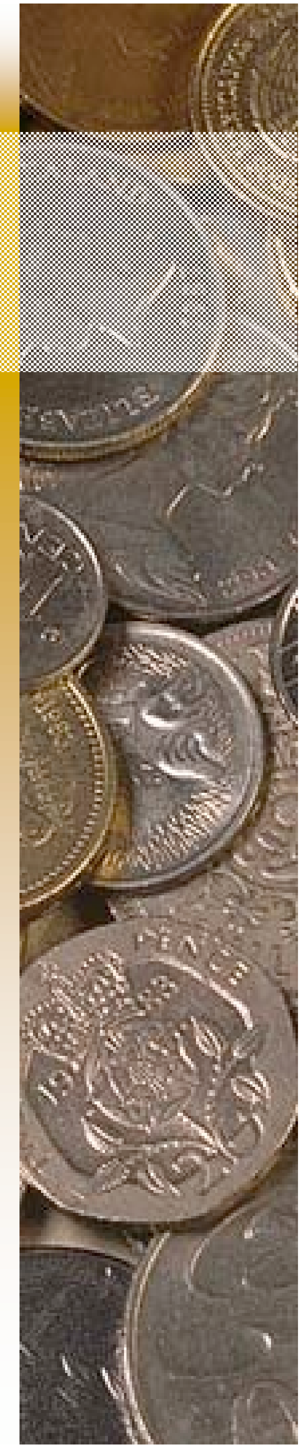
What are our conclusions regarding the CAPM?

- CAPM/SML concepts are based on expectations, yet betas are calculated using historical data. A company's historical data may not reflect investors' expectations about future riskiness.
- Other models are being developed that will one day replace the CAPM, but it still provides a good framework for thinking about risk and return.



What is the difference between the CAPM and the Arbitrage Pricing Theory (APT)?

- The CAPM is a single factor model.
- The APT proposes that the relationship between risk and return is more complex and may be due to multiple factors such as GDP growth, expected inflation, tax rate changes, and dividend yield.

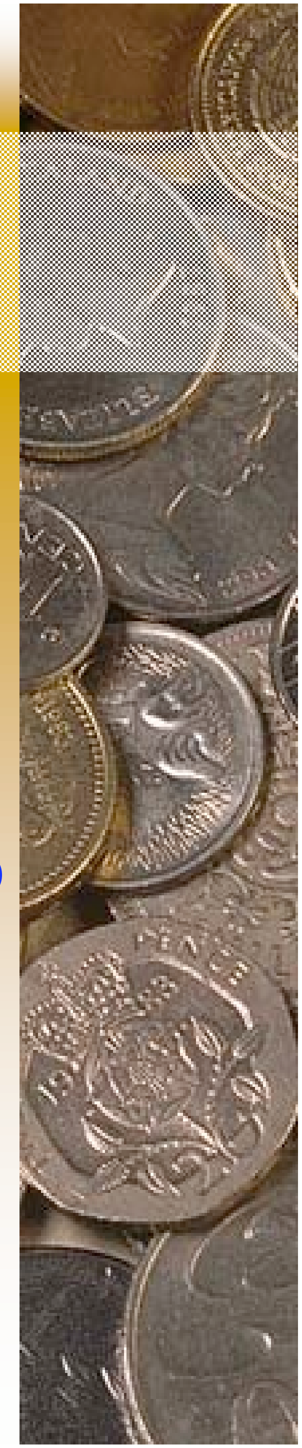


Required Return for Stock i under the APT

$$r_i = r_{RF} + (r_1 - r_{RF})b_1 + (r_2 - r_{RF})b_2 + \ldots + (r_j - r_{RF})b_j.$$

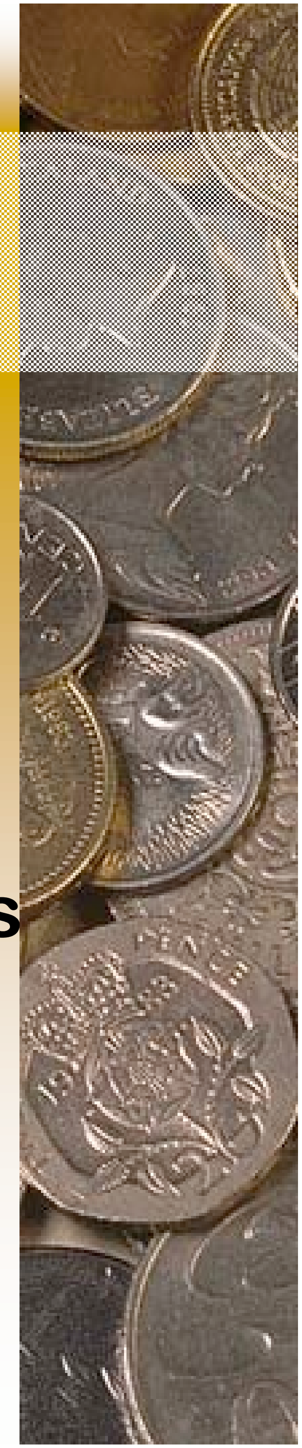
r_j = required rate of return on a portfolio sensitive only to economic Factor j.

b_j = sensitivity of Stock i to economic Factor j.

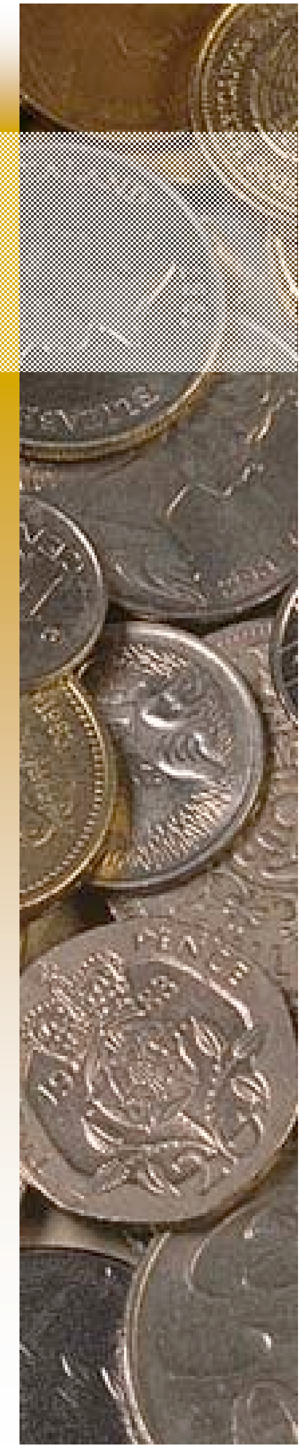


What is the status of the APT?

- The APT is being used for some real world applications.
- Its acceptance has been slow because the model does not specify what factors influence stock returns.
- More research on risk and return models is needed to find a model that is theoretically sound, empirically verified, and easy to use.



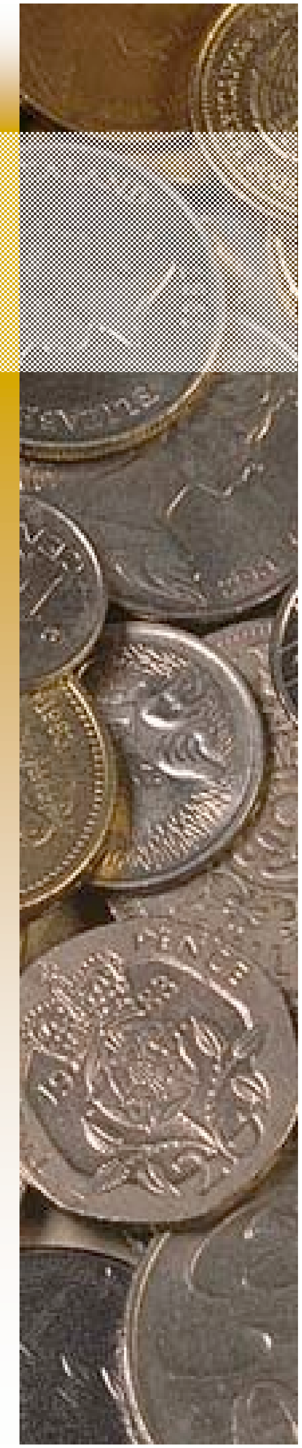
Fama-French 3-Factor Model



- Fama and French propose three factors:
  - The excess market return, r_M-r_{RF}.
  - the return on, S, a portfolio of small firms (where size is based on the market value of equity) minus the return on B, a portfolio of big firms. This return is called r_{SMB}, for S minus B.

Fama-French 3-Factor Model (Continued)

– the return on, H, a portfolio of firms with high book-to-market ratios (using market equity and book equity) minus the return on L, a portfolio of firms with low book-to-market ratios. This return is called r_{HML}, for H minus L.



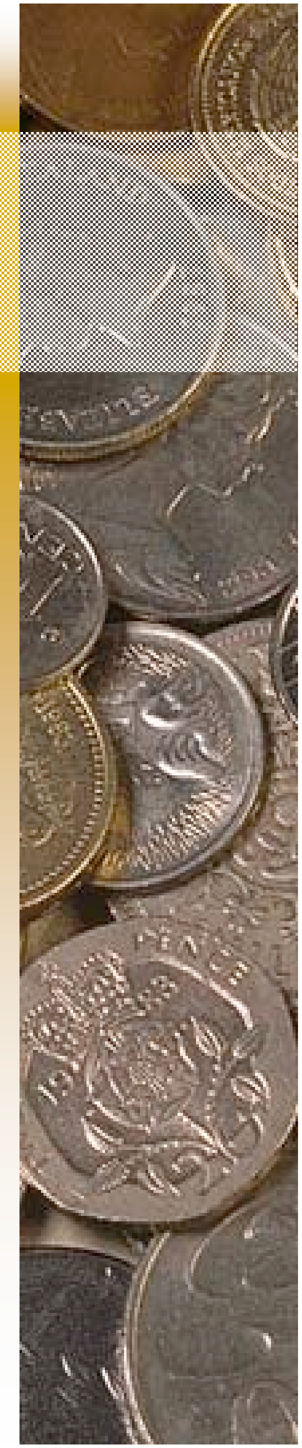
Required Return for Stock i under the Fama-French 3-Factor Model

$r_i = r_{RF} + (r_M - r_{RF})b_i + (r_{SMB})c_i + (r_{HMB})d_i$

b_i = sensitivity of Stock i to the market return.

c_j = sensitivity of Stock i to the size factor.

d_j = sensitivity of Stock i to the book-to-market factor.



Required Return for Stock i: b_i=0.9, r_{RF}=6.8%, the market risk premium is 6.3%, c_i=-0.5, the expected value for the size factor is 4%, d_i=-0.3, and the expected value for the book-to-market factor is 5%.

$$r_i = r_{RF} + (r_M - r_{RF})b_i + (r_{SMB})c_i + (r_{HMB})d_i$$

$$r_i = 6.8\% + (6.3\%)(0.9) + (4\%)(-0.5) + (5\%)(-0.3)$$

$$= 8.97\%$$

CAPM Required Return for Stock i

CAPM:

$r_i = r_{RF} + (r_M - r_{RF})b_i$

$r_i = 6.8\% + (6.3\%)(0.9)$

$= 12.47\%$

Fama-French (previous slide):

$r_i = 8.97\%$