1) (2 pts each, 16 pts total) For the tree below,

- **i)** Which nodes are the children of node 5?
- **ii)** Which nodes are the siblings of node 2?
- **iii)** Which nodes are internal?
- **iv)** Which nodes are the *proper* ancestors of node 11?
- **v)** What nodes are the descendants of node 21?
- **vi)** What is the height of the tree?
- **vii)** What would be the output of a pre-order traversal of the tree?
- **viii)** What would be the output of a level-order traversal of the tree?
2) (10 pts) Order the following by growth rate. Indicate if any are of the same complexity.

\[ n \quad e^n \quad n^2 \quad n \log n \quad \sqrt{n} \quad n \log (\log n) \quad 23 \quad n! \]

3) (8 pts each, 16 pts total) Suppose you are writing a Node class for a binary tree with the fields shown below:

```java
public class Node {
    int value;
    Node left, right;
}
```

a) Write the recursive method that would insert a new value into the node:

```java
public void add(int n) {
    // Method implementation
}
```

b) Write the recursive method returns a String representing a in-order traversal of the tree

```java
public String inOrderToString() {
    // Method implementation
}
```
4) (2 pts each, 12 pts total) What is the worst case $\Theta$ complexity for the operation findLargest() given the following implementations of a list of numbers:

<table>
<thead>
<tr>
<th></th>
<th>Worst Case Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unsorted LinkedList</td>
<td></td>
</tr>
<tr>
<td>Sorted LinkedList</td>
<td></td>
</tr>
<tr>
<td>Unsorted ArrayList</td>
<td></td>
</tr>
<tr>
<td>Sorted ArrayList</td>
<td></td>
</tr>
<tr>
<td>Binary Tree</td>
<td></td>
</tr>
<tr>
<td>Balanced Binary Tree</td>
<td></td>
</tr>
</tbody>
</table>

5) (7 pts) The time complexity of the Towers of Hanoi problem can be written recursively as $T(n) = 1 + 2T(n-1)$. Solve this recursion (e.g. using the method of telescoping).
6) (5 pts each, 15 pts total) Describe in words (and pictures) how the following sorts work.
   a) Insertion sort

   b) Merge Sort

   c) Quick Sort
7) (6 pts total) Suppose we *label* the nodes of a perfect binary tree according to their level-order as shown below. (The label is not the same thing as the item stored in the node). We find, for example, that the left child of the node labeled 2 is 5 and its right child is node 6.

[Diagram of a binary tree]

a) (2 pts) In general, given a node labeled $n$, what is the mathematical function that returns the index of its left child? That is, if I give you the label of a node, you must figure out the label of its left child.

b) (2 pts) Its right child?

c) (2 pts) Its parent?

8) (5 pts each, 10 pts total) A node in a tertiary tree has 3 children.
   a) Given a perfect tertiary tree, how many nodes would be in the $i$th level?

   b) How many nodes are in an entire perfect tertiary tree of height $h$?
9) (8 pts) Explain in words the minimax algorithm. Use an example or draw pictures to support your explanation.