1) (10 pts) The picture below shows the structure of the variable `cards`, which is a 2D array of `Card` objects. Write the java code necessary to create the structure shown below.
2) (2 pts each, 16 pts total) For the tree below,

i) Which nodes are the children of node 5?

ii) Which nodes are the siblings of node 11?

iii) Which nodes are internal?

iv) Which nodes are the proper ancestors of node 11?

v) What nodes are the descendants of node 5?

vi) What is the height of the tree?

vii) What would be the output of a post-order traversal of the tree?

viii) What would be the output of a level-order traversal of the tree?

3) (10 pts) Order the following by growth rate. Indicate if any are of the same complexity.

\[ n \quad (1000)^n \quad n^2 \quad n \log n \quad n^{\sqrt{n}} \quad n \log (n^2) \quad n^{1/4} \quad n! \]
4) (10 pts) Use mathematical induction to prove that \[ \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \].

5) (8 pts) Suppose you have a Node class for a binary search tree with the fields shown below:

```java
public class Node {
    int value;
    Node left, right;
}
```

Write a recursive method that returns the total number of nodes in a tree. For example, for the tree in problem 2, the method should return 11.

```java
public int getNumNodes() {
}
```
6) (14 pts total) Fibonacci
   a) (6 pts) Write a brute force recursive method that computes and returns the fibonacci numbers
      (assume that fib(0)=1, fib(1)=1, fib(n)=fib(n-1)+fib(n-2));

      public int fib(int n) {

      }

   b) (4 pts) Explain why this is not very efficient. Draw a picture if necessary.

   c) (4 pts) What is a technique that can be used to improve the efficiency? Explain how this
      technique could be applied to this problem.
7) (1 pt each, 25 pts total) Fill in the following table, except for the blocks that have XXXXX’s.

<table>
<thead>
<tr>
<th>Sort Algorithm</th>
<th>Average Complexity</th>
<th>Stable (yes/no)</th>
<th>Sorts in-place (yes/no)</th>
<th>Divide and conquer (yes/no)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insertion</td>
<td></td>
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<tr>
<td>Bubble</td>
<td></td>
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<tr>
<td>Stooge</td>
<td>XXXXX</td>
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<tr>
<td>Merge</td>
<td>XXXXX</td>
<td></td>
<td></td>
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<tr>
<td>Heapsort</td>
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<tr>
<td>Quick</td>
<td>XXXXX</td>
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<tr>
<td>Bucket (m buckets)</td>
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</tbody>
</table>


9) (1 pt each, 5 pts total) Heaps: What is the average time $\Theta$ complexity of each of the following operations when applied to a heap?
   a) build heap: ___________
   b) filter down: ___________
   c) remove largest: ___________
   d) look at largest: ___________
   e) change priority of an arbitrary node: ___________
10) (5 pts each, 15 pts total) Collision Detection: Assume you have a hash table for storing integers. It has a size of 10 and uses the standard "% tablesize" as the starting hash function. For each of the following collision detection strategies, show the resulting contents of the table after inserting the following items in the order given:

7, 3, 23, 5, 13, 27, 4

a) linear probing

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
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</table>

b) quadratic probing

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<thead>
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</table>

c) chaining

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</table>
11) (8 pts) Heaps: Given the array \(\{75, 80, 20, 41, 35, 4, 63, 30, 92\}\). Show the sequence of swaps that would be executed by the method buildheap (draw the heap as a tree, not as an array).