Summations

Summations can be finite or infinite. We define infinite sums as
\[ \sum_{k=1}^{\infty} f(k) = \lim_{n \to \infty} \sum_{k=1}^{n} f(k) \]

If we have a sum with asymptotic notation:
\[ \sum_{k=1}^{n} \Theta(f(k)) = \Theta(\sum_{k=1}^{n} f(k)) \]

Note that on left, \( \Theta \) is on \( k \), on right \( \Theta \) is on \( n \).

Known Series

- Arithmetic series
  This came up in insertion sort.
  \[ \sum_{k=1}^{n} k = 1 + 2 + 3 + \ldots + n = \frac{n(n + 1)}{2} \]
  Which is \( \Theta(n^2) \).

- Geometric series:
  1. Finite sum
     \[ \sum_{k=0}^{n} x^k = 1 + x + x^2 + \ldots + x^n = \frac{x^{n+1} - 1}{x - 1} \]
  2. Infinite sum
     \[ \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \ldots = \frac{1}{x - 1} \]
     for \( 0 \leq x < 1 \)

- Harmonic series (infinite sum blows up)
  \[ \sum_{k=0}^{\infty} \frac{1}{k} = \infty \]
  \[ \sum_{k=0}^{n} \frac{1}{k} = \ln n + O(1) \]

- Lots of sums are available in tables of mathematical functions.