Kleinberg and Tardos, Chapter 2, problem 6: Find the number of additions executed. Note, the summation below actually counts the number of terms added as opposed to the number of additions, e.g. \( A_1 + A_2 \) has 2 terms but one addition. However, the complexity will be the same and, besides, the exercise is more about doing summations.

\[
\sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=i}^{j} 1 = \sum_{i=1}^{n} \sum_{j=i+1}^{n} j - i + 1
\]

\[
= \sum_{i=1}^{n} \left[ \left( \sum_{j=i+1}^{n} j \right) + \left( (1 - i) \sum_{j=i+1}^{n} 1 \right) \right]
\]

\[
= \sum_{i=1}^{n} \left( \frac{n(n + 1)}{2} - \frac{i(i + 1)}{2} + (1 - i)(n - i) \right)
\]

After lots of simplification (just algebra!), we get

\[
\sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=i}^{j} 1 = \sum_{i=1}^{n} \left( \frac{i^2}{2} - \frac{(n + 3/2)i + n(n + 3)}{2} \right)
\]

\[
= \left( \frac{1}{2} \sum_{i=1}^{n} i^2 \right) - \left( \frac{n + 3/2}{2} \sum_{i=1}^{n} i \right) + \left( \frac{n(n + 3)}{2} \sum_{i=1}^{n} 1 \right)
\]

\[
= \left( \frac{1}{2} \frac{n(n + 1)(2n + 1)}{6} \right) - \left( n + 3/2 \frac{n(n + 1)}{2} \right) + \left( \frac{n(n + 3)}{2} \right)
\]

Lots more simplification gives the final result ...

\[
\sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=i}^{j} 1 = \frac{n}{6} \left( n^2 + 3n - 4 \right)
\]

To check the result, compare the value calculated directly from the summation with the value calculated from the above equation to see if they match. For example, when \( n = 3 \), we should get 7 (do you get this?)