1. (10 pts) State the definition of $f(n) = \Omega(g(n))$.

2. (12 pts) Use the definition you gave in the previous problem to prove or disprove the claim: $h_1(n) + h_2(n) = \Omega(max(h_1(n) + h_2(n)))$. Carefully explain all of your steps.
3. (12 pts) Order the following functions from smallest to largest based on their Big-Oh complexity. Be sure to identify which functions are the same (that is, which are Big-Theta of one another). Assume logs are base 2.

\[ n \log(n) \quad 100^2 \quad n! \quad n^3 \quad 2^n \quad \sqrt{n} \quad 2^{2 \log(n)} \quad \log^2(n) \]

4. (3 pts each, 18 pts total) Circle true or false:

(a) \( f(n) = \Omega(f(n)) \)  

(b) \( f(n) = \Omega(g(n)) \Rightarrow g(n) = O(f(n)) \)

(c) \( f(n) = O(g(n)) \Rightarrow f(n) = o(g(n)) \)

(d) \( f(n) = o(g(n)) \Rightarrow f(n) = O(g(n)) \)

(e) \( f(n) = O(g(n)) \Rightarrow \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \)

(f) \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \Rightarrow f(n) = \omega(g(n)) \)
5. (12 pts) Use induction to prove the following. Be sure to explain what you are doing in each step.

\[ \sum_{i=0}^{n} x^i = \frac{1 - x^{n+1}}{1 - x} \]

6. (12 pts) Suppose you are given an array containing \( n \) numbers. You call \textit{build-heap} on it so that the result is a heap. The largest item will be in the array at index 0. Give \textit{all} the possible places (i.e. array indices) where it would be possible to find the smallest item. Be specific and explain your answer. Show some examples in tree and/or array form to help explain your answer.
7. (12 pts) Describe at least two different collision detection methods for hash tables.

8. (12 pts) Develop an algorithm that computes the $k^{th}$ largest element of a set of $n$ distinct integers in $O(n + k \log n)$ time.

9. (0 pts) What would the answer to this question be if you didn’t already know the answer?