1) (5 pts total) Discuss the meaning of and motivation for homogeneous coordinates.

Homogeneous coordinates allow one to represent coordinate systems that differ by orientation and by the location of their origin. The motivation is:
1) Translations can be represented by matrix multiplication thus giving it the same form as scaling and rotation.
2) Vectors and points have different representations. Specifically, their fourth coordinate is different: vectors \((v_x, v_y, v_z, 0)\) and points \((p_x, p_y, p_z, 1)\). This reflects that fact that points are fixed in space (depend on the origin) and vectors are not – only depend on direction.

2) (5 pts) What is Gouraud shading and how is it different from Phong shading? What is the advantage/disadvantage of each?

Gouraud shading computes the lighting/color values at only the vertices of surfaces. The lighting/color of other points on the surface are computed by interpolating the values at the vertices.
Pros: this is computationnally cheap
Cons: lighting effects that are highly location and orientation dependent, e.g. specular lighting are very poorly modeled

Phong shading computes the normals at the vertices. The lighting/color is computed for all points on the surface by first interpolating the normals and then calculating the light/color based on the normal at each specific surface location.
Pros: lighting effects that are highly location and orientation dependent look much better
Cons: it is more computationally expensive.
3) (5 pts) Given a triangle in 3D space with vertices $V_0$, $V_1$, and $V_2$. How do you calculate the normal to the triangle surface?

Let:

\[
\begin{align*}
a &= v_1 - v_0 \\
b &= v_2 - v_0
\end{align*}
\]

Then the surface normal is $a \times b$

4) (5 pts) How are shadows computed in ray tracing? Please use complete sentences and be sure to define terms. Include picture.

A ray is sent from the camera in the direction of a pixel. The point of first intersection with an object is called the intersection point. From this point, a ray is sent in the direction of a light source. If an object is encountered before reaching the light source, then the intersection point is in shadow with respect to this light source.

5) (5 pts) If you apply the convolution filter

\[
\begin{pmatrix}
-2 & 0 & 0 \\
0 & 4 & 0 \\
0 & 0 & -2
\end{pmatrix}
\]

to an image, what will be the characteristics of the resulting image? Explain your answer.

Solid areas in the picture would turn black. Diagonal lines (\(\backslash\)) would be invisible but diagonal lines the other way (\(/\)) would be visible. Horizontal and vertical lines and edges would be brighter.
6) (5 pts each, 15 pts total) Suppose you want to apply the following texture to the square surface on the right.

If the code below is applied, the square will be textured as shown below on the right.

```c
#include <GL/gl.h>

// Set texture parameters
glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_WRAP_S, GL_REPEAT);
glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_WRAP_T, GL_REPEAT);

// Draw the square
glBegin(GL_POLYGON);
    glVertex3f(v0x, v0y, 0);
    glVertex3f(v1x, v1y, 0);
    glVertex3f(v2x, v2y, 0);
    glVertex3f(v3x, v3y, 0);
glEnd();
```

In the rectangle on the right, sketch how the texture would appear if the following texture coordinates are used instead?

a. ```c
    glTexCoord3f(0.5, 0); glVertex3f(v0x, v0y, 0);
    glTexCoord3f(1, 0); glVertex3f(v1x, v1y, 0);
    glTexCoord3f(1, 1); glVertex3f(v2x, v2y, 0);
    glTexCoord3f(0.5, 1); glVertex3f(v3x, v3y, 0);
```  

b. ```c
    glTexCoord3f(0, 1); glVertex3f(v0x, v0y, 0);
    glTexCoord3f(1, 1); glVertex3f(v1x, v1y, 0);
    glTexCoord3f(1, 0); glVertex3f(v2x, v2y, 0);
    glTexCoord3f(0, 0); glVertex3f(v3x, v3y, 0);
```  

c. ```c
    glTexCoord3f(0, 0); glVertex3f(v0x, v0y, 0);
    glTexCoord3f(2, 0); glVertex3f(v1x, v1y, 0);
    glTexCoord3f(2, 1); glVertex3f(v2x, v2y, 0);
    glTexCoord3f(0, 1); glVertex3f(v3x, v3y, 0);
```
7) (3pts each, 18 pts total) Blending in OpenGL: Suppose that you draw two rectangles $S_1$ and $S_2$, using OpenGL where
- $S_1$ is closer to the camera than $S_2$.
- $S_1$ is drawn first followed by $S_2$.
- The background color is set to white=$(1,1,1)$.
- the `glColor` is set to green=$(0,1,0,\alpha_1)$ when $S_1$ is drawn.
- the `glColor` is set to red=$(1,0,0,\alpha_2)$ when $S_2$ is drawn.

What is the resulting $(r,g,b)$ color in the frame buffer corresponding to each of the following. Explain how you calculated your answer (you will not get full credit for just an answer written down)

a. Area 2 if $\alpha_1=0.5$, $\alpha_2=0.25$ and z-buffering is turned on.

$$ (r,g,b) = ____{(1,.75,.75)} ____ = .25(1,0,0) + .75*(1,1,1) = \alpha_2 \text{ color}(S_2) + (1-\alpha_2) \text{ color}(bg) $$

b. Area 1 if $\alpha_1=1.0$, $\alpha_2=1.0$ and z-buffering is turned on.

$$ (r,g,b) = ____{(0,1,0)} ____ = \text{ color}(S_1) $$

$S_1$ is opaque and is in front. With z-buffering on, $S_2$ won't be seen.

c. Area 1 if $\alpha_1=1.0$, $\alpha_2=1.0$ and z-buffering is turned off.

$$ (r,g,b) = ____{(1,0,0)} ____ = \text{ color}(S_2) $$

Since z-buffering is off, whatever is drawn second ($S_2$) is on top. Since $\alpha_2=1$, it is opaque.

d. Area 1 if $\alpha_1=0.25$, $\alpha_2=1.0$ and z-buffering is turned on.

$$ (r,g,b) = ____{(0.75, 1, .75)} ____ = .25(0,1,0) + .75*(1,1,1) = \alpha_1 \text{ color}(S_1) + (1-\alpha_1) \text{ color}(bg) $$

Since z-buffering is on and $S_1$ drawn first and is in front, $S_2$ has no effect in area 1

e. Area 1 if $\alpha_1=1.0$ $\alpha_2=0.25$ and z-buffering is turned off.

$$ (r,g,b) = ____{(0.25,0.75,0)} ____ = \alpha_2 \text{ color}(S_2) + (1-\alpha_2) \text{ color}(S_1) = .25(1,0,0) + .75*(0,1,0) $$

f. Area 1 if $\alpha_1=0.5$, $\alpha_2=0.5$ and z-buffering is turned off.

$$ (r,g,b) = _{_{_{0.5}}}(.75, .5, .25) ____ = \alpha_2 \text{ color}(S_2) + (1-\alpha_2) \left( \alpha_1 \text{ color}(S_1) + (1-\alpha_1) \text{ color}(bg) \right) \\
\phantom{_____{_{_{0.5}}}}.5 (1,0,0) + .5 \left( .5 (0,1,0) + .5 (1,1,1) \right) \\
\phantom{_____{_{_{0.5}}}}(.5,0,0) + (0, .25, 0) + (.25, .25, .25) $$
8) (6 pts each, 42 pts total) For each of the coordinate systems in the rendering pipeline: 1) what is the meaning and motivation of the coordinate system and 2) describe the transformations or process required to obtain it from the previous coordinate system. Please use complete sentences and include as many specifics as you can remember. Include pictures to clarify the explanation. (Don’t rush, you have lots of time!)

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a. Modeling Coordinates:

b. World Coordinates & Modeling Transformation:
c. Eye/Camera Coordinates & Viewing Transformation:

d. Clip Coordinates & Projection Transformation:

e. Normalized Device Coordinates & Perspective Division:
f. Window Coordinates & Viewport Transformation:

g. 2D Screen Coordinates & Rasterization: