Group Exam 4

Name: _____

Name of group member: _____

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Problem 3: A tank holds 5000 gallons of water which drains from a hole in the bottom of the tank in 40 minutes. Torricelli's Law gives that the volume of water remaining in the tank after t minutes is given by

$$V = 5000(1 - \frac{t}{40})^2 \qquad 0 \leq t \leq 40$$

(a) Find the rate at which water is draining from the tank after 5min.

(b) Find the rate at which water is draining from the tank after 10 min.

(c) Find the rate at which water is draining from the tank after 20 min.

(c) Find the rate at which water is draining from the tank after 40 min.

(d) At what time is the water flowing out the fastest? The slowest? Summarize your findings.

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Problem 1: (a) Write $|x| = \sqrt{x^2}$ and use the Chain Rule to show that

$$\frac{d}{dx}\Big(|x|\Big) = \frac{x}{|x|}$$

.

(b) If $f(x) = |\sin(x)|$, find $f'(x)$ and sketch the graphs of f and f'.

(c) If $g(x) = \sin(|x|)$, find $g'(x)$ and sketch the graphs of g and g'.

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Problem 2: The definition of the inverse tangent function, which is called the *arctangent* function, is given by

$\arctan(x) = y$	means	$\tan(y) = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$

(a) Follow the steps below to find the derivative of the $\arctan(x)$ function.

Let $y = \arctan(x)$, then we are after a formula for $\frac{dy}{dx}$. By the definition of $\arctan(x)$ given above, $-\frac{\pi}{2} < y < \frac{\pi}{2}$ and

$$\tan(y) = x$$

Take the derivative implicitly of the formula above to find a formula for $\frac{dy}{dx}$. (Note, your answer will have a y in it, but we want a formula with only x's in it.)

(b) Label all three sides of the right triangle below to reflect the fact that $\tan(y) = x$.

(c) Recall that the $\sec(y) = \frac{1}{\cos(y)}$. Use the picture above to find an expression for $\sec(y)$ in terms of x.

(d) Substitute your expression from part (d) into the formula for $\frac{dy}{dx}$ that you found in part (b).