1. (Definition of derivative) Use the definition of the derivative $f'$ to find derivative of

(a) $f(x) = \sqrt{x}$ at the given $x = 1$.
(b) $f(x) = x^3$ at $x = 2$.
(c) $f(x) = \frac{1}{\sqrt{x}}$ at the given $x = 1$.
(d) $f(x) = \sqrt{4x+3}$ at any $x > -\frac{3}{4}$.

2. (A meaning of derivative, slope of tangent line) Consider the graph of $f(x) = x^3$. Find the equation of the tangent line at the point $(2, 8)$ on the graph.

3. (Differentiability implies continuity. But, not the converse) Find an example of function which is continuous at $x = 0$, but not differentiable at $x = 0$.

4. (Power rule) Use power rules to find the derivatives $f'$ of

(a) $f(x) = x^3 - x^2 - 5x + 8 - \frac{1}{2x}$.
(b) $f(x) = 3x^5 + \frac{3}{\sqrt{x}}$.
(c) $f(x) = 1 - x^2 + 3x^3\sqrt{x}$.

5. (Derivative, rate of change, velocity) On top of a cliff with height 73.5 m we throw a ball upward. The height of the ball from the ground follows the function $h(t) = 73.5 + 9.8t - 4.9t^2$.

(a) Find the maximum height the ball can reach.
(b) Find the velocity when the ball hits the ground.

6. (Derivative, rate of change, velocity) A particle is moving along a line with displacement function $s(t) = t^3 - 6t^2 + 9t$.

(a) When are the moments the particle change its direction.
(b) Find the total distance the particle traveled in $t = 4$.

7. (Derivative of $e^x$) Find the derivative $f'$ of the function $f(x) = e^{x+1} - 2x + \ln 2$.

8. (Product rule, Quotient rule) Find the derivatives $f'$ of the following functions.

(a) $f(x) = (x^2 - 3x + 1)(3x + 2)$.
(b) $f(x) = e^x(\sqrt{x} + 5x^3)$.
(c) $f(x) = \frac{x^4 + 2}{\sqrt{x} - 2}$ at $x = 1$.
(d) $f(x) = \frac{e^x}{\sqrt{x} - 2}$.
9. **(Derivatives of trigonometric functions) Find the derivatives \( f' \) of the following functions.**

   (a) \( f(x) = \sin x + \tan x - \sec x \).

   (b) \( f(x) = \sec x \tan x \) at \( x = \frac{\pi}{3} \).

   (c) \( f(x) = xe^x \sin x \).

   (d) \( f(x) = \frac{\tan x - 1}{\sec x} \) at \( x = \frac{\pi}{3} \).

10. **(An important limit) Taking \( f(x) = \sin x \), we observed**

    \[
    1 = \cos 0 = f'(0) = \lim_{x \to 0} \frac{\sin x - \sin 0}{x - 0} = \lim_{x \to 0} \frac{\sin x}{x}.
    \]

    Use above identity to find the limit of

    \[
    \lim_{x \to 0} \frac{\sin(4x)}{3x}.
    \]

11. **(Chain rule) Find the derivatives \( f' \) of the following functions.**

    (a) \( f(x) = (3x^3 - 2x^2 + 5)^{31} \).

    (b) \( f(x) = \sqrt{1 - x^2} \).

    (c) \( f(x) = \sin(x^3) \).

    (d) \( f(x) = \sec^2 x - \tan^2 x \).

    (e) \( f(x) = (\sin(\tan x))^3 \).

    (f) \( f(x) = e^{\cos x} \).

    (g) \( f(x) = 2^{-x^2} \).

    (h) \( f(x) = x \sin \left(\frac{1}{x}\right) \) for \( x > 0 \).

12. **(Implicit differentiation) Consider the graph of \( \sqrt{x} + \sqrt{y} = 1 \) in the \( xy \)-plane. Find the equation of the tangent line at the point \((\frac{1}{4}, \frac{1}{4})\) on the graph.**

13. **(Implicit differentiation) Find the equation of the tangent line to \( x^3 + y^3 = 4xy \) at the point \((2, 2)\).**

14. **(Implicit differentiation) Find the equation of the tangent line to \( 2(x^2 + y^2)^2 = 25(x^2 - y^2) \) at the point \((3, 1)\).**

15. **(Shapes of the inverse trigonometric functions) Sketch the graph of inverse trigonometric functions \( y = \sin^{-1} x \), \( y = \cos^{-1} x \), \( y = \tan^{-1} x \). What are the domains and ranges.**

16. **(Derivatives of inverse trigonometric functions) Find the derivatives \( f' \) of the following functions.**

    (a) \( f(x) = x \arctan \sqrt{x} \).

    (b) \( f(x) = \sin^{-1}(x^3) \).
Math 141, Test 2, practice problems

(Answer keys)

1. (a) \( f'(1) = \frac{1}{2} \)  (b) \( f'(2) = 12 \)  (c) \( f'(1) = -2 \)  (d) \( f'(x) = \frac{2}{\sqrt{4x+1}} \)  

2. \( y = 12x - 16 \)

3. \( f(x) = |x| \)

4. (a) \( f'(x) = 3x^2 - 2x - 5 + \frac{5}{x^2} \)  (b) \( f'(x) = 15x^4 - \frac{1}{x^3} \)  (c) \( f'(x) = -2x + \frac{21}{2}x^{5/2} \)

5. (a) 78.4 m  (b) -39.2 m/s

6. (a) \( t = 1 \), \( t = 3 \)  (b) 12

7. \( f'(x) = e^{x+1} - 2 \)

8. (a) \( f'(x) = 9x^2 - 14x - 3 \)  (b) \( f'(x) = e^x(\sqrt{x} + 5x^3 + \frac{1}{2}\sqrt{x} + 15x^2) \)  (c) \( f'(x) = \frac{e^{x(4-x)}}{(3-x)^2} \)

9. (a) \( f'(x) = \cos x + \sec^2x - \sec x \tan x \)  (b) \( f'(\frac{x}{3}) = 14 \)

(c) \( f'(x) = e^x(\sin x + x \sin x + x \cos x) \)  (d) \( f'(\frac{x}{3}) = \frac{1-e^x}{2} \)

10. \( \frac{4}{x} \)

11. (a) \( f'(x) = 331(3x^3 - 2x^2 + 5)^{330}(9x^2 - 4x) \)  (b) \( f'(x) = \frac{x}{\sqrt{4-x^2}} \)  (c) \( f'(x) = 3x^2 \cos(x^3) \)

(d) \( f'(x) = 0 \)  (e) \( f'(x) = 3(\sin(tan x))^2 \cos(tan x) \sec^2 x \)  (f) \( f'(x) = -\sin x e^{\cos x} \)

(g) \( f'(x) = -2x^2 - x^2 \)  (h) \( f'(x) = \sin \left(\frac{1}{x}\right) - \frac{1}{x} \cos \left(\frac{1}{x}\right) \)

12. \( y = -x + \frac{1}{2} \)

13. \( y = -x + 4 \)

14. \( y = -\frac{9}{11}x + \frac{40}{11} \)

15. Lecture note

16. (a) \( f'(x) = \arctan \left(\sqrt{\frac{x}{2(1+x)}}\right) \)  (b) \( f'(x) = \frac{3x^2}{\sqrt{1-x^3}} \)