MATH 251 Midterm Exam #1
February 11, 2005

NAME: ______________________________ (please print)

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
total	100	

1. a) Prove that there exist integers m and n such that $2m + 7n = 1$.

   b) Prove that there do not exist integers m and n such that $12m + 15n = 1$.

2. Let A, B, and Q be statements. Prove the following equivalence.

   $(A \wedge B) \rightarrow Q \Leftrightarrow A \rightarrow (B \rightarrow Q)$

3. Consider the statement below.

   There is a flying saucer which is aiming to conquer all galaxies.

   (a) Convert the statement into a statement with explicit quantifiers.

   (b) Negate the statement and simplify your negation.

   (c) Convert the simplified negated statement back into English words which best describe its meaning.

4. Write a derivation for the following argument. Determine whether or not the premises are consistent.

   Megan likes cats and dogs, but doesn't like bugs. If Megan likes porcupines, then she doesn't like cats. Megan likes porcupines or horses. If Megan likes bugs or snakes, then she doesn't like horses. Therefore Megan doesn't like snakes.

5. Analyze the alleged proof given below and give it one of three grades.

   Assign a grade of A if the claim and proof are correct, even if the proof is not the simplest or the proof you would have given.

   Assign a grade of F if the claim is incorrect, if the main idea of the proof is incorrect, or if most of the statements are incorrect.

   Assign a grade of C if the proof is largely correct but contains one or two incorrect statements or justifications.

   Whenever the proof is incorrect **explain what is incorrect and why**. Give justification for your grade.

   Let m be an integer.

   <u>Claim</u> If m^2 is odd, then m is odd.

   <u>"proof"</u> Assume that m^2 is not odd. Then m^2 is even and $m^2 = 2k$ for some integer k. Thus $\sqrt{2k}$ is an integer. If $\sqrt{2k}$ is odd, this implies $\sqrt{2k} = 2q + 1$ for some integer q, which means $m^2 = 2k = (2q + 1)^2 = 4q^2 + 4q + 1 = 2(2q^2 + 2q) + 1$. Thus m^2 is odd which contradicts our assumption that m^2 is not odd. Therefore $\sqrt{2k} = m$ must be even. Thus if m^2 is not odd, then m is not odd. Hence if m^2 is odd, then m is odd.