MATH 251 Midterm Exam #2
March 16, 2005

NAME: ______________________________ (please print)

You do NOT have to copy the statement of problem. You do NOT have to work the problems in order. However, please clearly label your solutions.

Problem	Points	Score
1	25	
2	25	
3	30	
4	20	
total	100	

1. Definitions. Give a precise definition of each of the following terms in bold below.

   Let $f : A \to B$ be a function.

   (a) Define what it means for g to be a **right inverse** of f.

   (b) Define what it means for g to be a **left inverse** of f.

   (c) Let $f : [0, 9] \to [0, \infty)$ be the function defined by $f(x) = \sqrt{x}$ for all $x \in [0, 9]$. Find two distinct left inverses of f.

2. Calculations.

   Let $A = \{1, 2, 3, 4, 5\}$ and $C = \{x, y, z, w, v\}$

   (a) Suppose $\pi_2 : A \times C \to C$ is the projection function. Calculate $(\pi_2)^*(\{x, y\})$.

   (b) Suppose $f : A \to C$ is given by $f(1) = y,\ \ f(2) = z,\ \ f(3) = x,\ f(4) = y,\ f(5) = x$. Calculate $f_*(A) - f_*(\{2, 3\})$.

   Suppose $B_i = (-5 - \frac{2}{i}, 6 + i)$. Suppose $I = \mathbf{N}$ and $J = \{2, 3, 4\}$ are indexing sets.

   (c) Calculate $\bigcup_{i \in I} B_i$ and $\bigcap_{i \in I} B_i$.

   (d) Calculate $\bigcup_{i \in J} B_i$ and $\bigcap_{i \in J} B_i$

3. Proofs.

   Let $f : A \to B$ be a function. Let $V \subseteq B$ and $W \subseteq B$.

   Prove that $f^*(W \cup V) = f^*(W) \cup f^*(V)$.

4. Proof Comprehension.

   Analyze the alleged proof given below and give it one of two grades.

   Assign a grade of A if the claim and proof are correct, even if the proof is not the simplest or the proof you would have given.

   Assign a grade of F if the claim is incorrect, if the main idea of the proof is incorrect, or if most of the statements are incorrect.

   If the proof is incorrect **explain exactly which part is incorrect and why**. Give justification for your grade.

   Claim Let A, B and C be sets. If $A \subseteq B \cup C$, then $A \subseteq B$ or $A \subseteq C$.

   proof Suppose $A \subseteq B \cup C$. Let $x \in A$. Since $A \subseteq B \cup C$ then $x \in B \cup C$. Hence $x \in B$ or $x \in C$. We break this into two cases. If $x \in B$ then we have shown $x \in A$ implies $x \in B$. If $x \in C$ then we have shown $x \in A$ implies $x \in C$. Therefore we have proven if $A \subseteq B \cup C$, then $A \subseteq B$ or $A \subseteq C$.