

MATH 251 Midterm Exam #1
February 17, 2006

NAME: _____ (please print)

Problem	Points	Score
1.1	10	
1.2	15	
2.1	15	
2.2	15	
2.3	10	
3	35	
total	100	

Part I: Precise statements of definitions and True/False questions.

(1.1) Give the precise definition of $a|b$. [10 pts]

(1.2) True or False. If false, give a counterexample. [15 pts]

- (a) If $x + y$ is an integer, then either x or y is an integer.
- (b) For all real numbers y , there exists a real number x such that $x^2 + y = 4$.
- (c) If n is an integer, then $n(n + 1)$ is even.

Part II: Short Answer.

(2.1) Consider the statement below: [15 pts]

If $|x| < 4$ and y is pink, then $|x - y| \geq 6$ or pigs fly over the moon.

- (a) Write the converse of the above statement. (Simplify all negations if there are any.)
- (b) Write the contrapositive of the above statement. (Simplify all negations if there are any.)
- (c) Suppose you wanted to prove this statement by contradiction. Write down all that you would assume to be true.

(2.2) Consider the statement below. [15 pts]

Every person in this class like to eat some kind of fruit.

- (a) Convert the statement into a statement with explicit quantifiers.
- (b) Negate the statement and simplify your negation.
- (c) Convert the simplified negated statement back into English words which best describe its meaning.

(2.3) Find $\gcd(105, 135)$ and write it as a linear combination of 135 and 105. [10 pts]

Part III. Proofs. Do **TWO** of the following four proofs. Make is clear which two proofs you are writing. [Do NOT DO THREE OR ALL FOUR of the following problems. You will not earn extra credit for doing more.] Please include your scratch work in addition to a formal proof. [35 pts]

(3.1) Prove that given positive real numbers a, b, c and d , if $a < b$ and $c < d$, then $ac < bd$.

(3.2) Let n be an integer. Prove that if n^2 is odd then, then n is odd.

(3.3) Suppose a, b, c and d are integers. Prove if $a|b$ and $c|d$ then $ac|bd$.

(3.4) Prove that for all real numbers x , $x \cdot 0 = 0$.