

Part 1: Definitions and Statements of Properties and Theorems.

1.1 Suppose $S \subseteq \mathbb{R}$ is a bounded set. State the precise definition of $\sup(S)$.

1.2 Fill in the blanks.

Order Properties of \mathbb{R} There is a nonempty subset \mathbb{P} , called the set of **positive real numbers**, that satisfies the following properties:

(i) _____

(ii) _____

(iii) _____

1.3 Let $X = (x_n)$ be a sequence of real numbers. State the precise definition of $\lim(x_n) = x$.

Part 2: Calculations.

2.1 Find the solution set of x -values which satisfy $\left| \frac{2+x}{3-x} \right| \geq 1$.

2.2 Find the infimum of $S := \left\{ \frac{2}{n} - \frac{3}{m} : n, m \in \mathbb{R} \right\}$. Prove your calculation is correct.

Part 3: Proofs

3.1 Use the Algebraic Properties of \mathbb{R} to prove: If $a \neq 0$ in \mathbb{R} , then $1/(1/a) = a$. [Note, the Algebraic Properties of \mathbb{R} are given below.]

3.2 Prove $\lim \left(\frac{2n}{n + 4\sqrt{n}} \right) = 2$

(end of Exam.)

Algebraic Properties of \mathbb{R} On the set \mathbb{R} of real numbers there are two binary operations, denoted by $+$ and \cdot and called addition and multiplication, respectively. These operations satisfy the following properties:

(A1) $a + b = b + a$ for all $a, b \in \mathbb{R}$,

(A2) $(a + b) + c = a + (b + c)$ for all $a, b, c \in \mathbb{R}$,

(A3) there exists an element 0 in \mathbb{R} such that $0 + a = a$ for all a in \mathbb{R} ,

(A4) for each a in \mathbb{R} there exists an element $-a$ in \mathbb{R} such that $a + (-a) = 0 = (-a) + a$,

(M1) $a \cdot b = b \cdot a$ for all $a, b \in \mathbb{R}$,

(M2) $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ for all $a, b, c \in \mathbb{R}$,

(M3) there exists an element 1 in \mathbb{R} *distinct from 0* such that $1 \cdot a = a = a \cdot 1$ for all $a \in \mathbb{R}$,

(M4) for each $a \neq 0$ in \mathbb{R} there exists an element $1/a$ in \mathbb{R} such that $a \cdot (1/a) = 1 = (1/a) \cdot a$.

(D) $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(b + c) \cdot a = b \cdot a + c \cdot a$ for all $a, b, c \in \mathbb{R}$.