Math 251: Foundations of Advanced Mathematics
Individual Problems #5

Prove the following theorems.

1. Suppose that A and B are finite sets, A has m elements, B has n elements, and $f : A \to B$ is a function. The following are equivalent.
   (a) f is one-to-one and onto.
   (b) f is one-to-one and $m = n$.
   (c) f is onto and $m = n$.
2. Let A be a totally ordered set, and let (s_i) be a sequence in A.
   (a) **Theorem 5.5.21.1.** If (s_i) is constant, then every subsequence of (s_i) is constant.
   (b) **Theorem 5.5.21.2.** If (s_i) has distinct terms, then every subsequence of (s_i) has distinct terms.
   (c) **Theorem 5.5.22.** If (s_i) is increasing, then every subsequence of (s_i) is increasing.