Math 251: Foundations of Advanced Mathematics
Written Problems #4

1. A function $f : \mathbb{R} \to \mathbb{R}$ is ***increasing*** if $a < b$ implies $f(a) < f(b)$ for all $a, b \in \mathbb{R}$. Prove that if f is increasing, then f is one-to-one.

2. (Exercise 4.3.6) Suppose $f : A \to B$ and $g : B \to C$ are functions which have inverses f^{-1} and g^{-1}. Prove that the inverse function of $g \circ f$ is $f^{-1} \circ g^{-1}$.

3. (Exercise 4.4.11) Let $f : A \to B$ be a function. Prove the following statements.

   (a) The function f is one-to-one if and only if $E = f^*(f_*(E))$ for all subsets $E \subseteq A$.

   (b) The function f is onto if and only if $F = f_*(f^*(F))$ for all subsets $F \subseteq B$.