Math 476: Modern Geometry
Individual Problems #1

1. The Pythagorean Theorem is the theorem in Euclidean geometry with more proofs than any other. There have been literally hundreds of different proofs of the Pythagorean Theorem over the years. Here's one version of the theorem:

   Theorem. *Suppose that $\triangle ABC$ is a triangle, $\angle ABC$ is a right angle, the length of $\overline{AB}$ is a, the length of $\overline{BC}$ is b, and the length of $\overline{AC}$ is c. Then $a^2 + b^2 = c^2$.*

   I have attached 3 proofs of this theorem from the series of books, "Proofs Without Words," by Roger Nelson of Lewis and Clark College. These "proofs" are really diagrams that are supposed to indicate proofs. Fill in the actual proofs that accompany these diagrams. (Note: These proofs use the rigorous logic of Euclidean geometry, but assume a little more than we've been assuming, for example, that the area of a triangle with base b and height h is $\frac{1}{2}bh$.)

2. Given positive integers b, v, r, k, and λ, a ***block design*** is a set of v points and b lines (or "blocks") such that each point appears on exactly r lines, each line contains exactly k points, and each pair of points appears together on exactly λ lines. The study of block designs is an active research area in combinatorics.

   (a) Show that a finite projective plane of order n is a block design. Express the numbers b, v, r, k, and λ in terms of n.

   (b) A block design is ***resolvable*** if the lines can be grouped into s sets each of which partitions the set of points. Show that a finite affine plane of order n is a resolvable block design. Express the numbers b, v, r, k, λ, and s in terms of n.

   (c) There are block designs that are not projective planes or affine planes. Find a block design with $b = 7$, $v = 7$, $r = 4$, $k = 4$, and $\lambda = 2$. (Note: It may be easier to list your "lines" as sets of points than to try to draw them in a diagram.)