Math 476: Modern Geometry
Individual Problems #2

1. The ***Poincaré upper half plane*** is another model of hyperbolic geometry.

hyperbolic term	interpretation
plane	the set of points in the Euclidean plane with $y > 0$
point	a point in the plane with $y > 0$
line	either the part of a vertical line with $y > 0$ or the top half of a circle with center on the x-axis

   (a) Verify that this is a valid model of hyperbolic geometry by checking that the five hyperbolic axioms hold. Write an argument for axioms 1 and 5, and convince yourself informally of the others.

   (b) Repeat Exercise 2.8.1 for this model.

2. This problem outlines an alternate definition of *sensed parallel* in hyperbolic geometry that does not require the concepts of "betweenness" or "clockwise" versus "counter-clockwise" that we used to define sensed parallel lines.

   (a) Informally justify the following statement using the Poincaré disc model: Three lines are mutually sensed parallel if and only if for every point P on any of them, there exists a unique fourth line through P parallel to the other two.

   (b) Given two parallel lines, explain how to use part (a) to determine if they're sensed parallel.

3. Given a line l, let R_l denote reflection through l. Prove the following statements.

   (a) If T is a translation along the line l and m is a line perpendicular to l, then there is a unique line n perpendicular to l such that $T = R_m R_n$.

   (b) If R is a rotation about the point P, and l is a line through P, then there is a unique line m through P such that $R = R_l R_m$.