Math 399 – Topics in Graph Theory
Hamiltonian Cycles Handout

A *Hamiltonian cycle* is a cycle which contains every vertex in the graph. A graph is *Hamiltonian* if it contains a Hamiltonian cycle.

1. The *icosahedron graph* is shown below. Although he was not the first to study Hamiltonian cycles, their namesake Sir William Rowan Hamilton popularized them by looking for Hamiltonian cycles on the icosahedron graph. He even published a board game in which players searched for Hamiltonian cycles in the icosahedron graph. The game was not commercially successful. Show that the icosahedron graph is Hamiltonian.

2. The *product* of two graphs G and H, denoted $G \times H$, is the graph with vertex set $\{(x, y)|x \in G, y \in H\}$ and with an edge between (x_1, y_1) and (x_2, y_2) if and only if either ($x_1 = x_2$ and $y_1 \sim y_2$), or ($y_1 = y_2$ and $x_1 \sim x_2$). We denote a path on n vertices by P_n. Draw the graph $P_3 \times P_4$. Is this graph Hamiltonian?

3. Consider a 4×4 chessboard with two opposite corners removed, leaving 14 squares. We place dominos on the board, so that each domino covers two adjacent squares. Is it possible to place 7 dominos in this way to completely cover the board?

4. Can an 8×8 chessboard with two opposite corners removed be completely covered with 31 dominos in the same way?

5. A mouse is eating a 3-inch cube of cheese. It decides to divide the cube into 27 1-inch cubes, and eat these one at a time. It starts at a corner cube, and always moves on to an adjacent cube. It wants to eat all 27 1-inch cubes, ending with the center cube. Is this possible? (Ignore gravity.)

6. For which positive integers n is $P_3 \times P_n$ Hamiltonian?

7. For which positive integers m and n is $P_m \times P_n$ Hamiltonian?

8. For which positive integers m, n, and k is $P_m \times P_n \times P_k$ Hamiltonian?