

## Math 399 – Topics in Graph Theory

### Graph Pebbling Handout

A *pebbling distribution* is a placement of pebbles on the vertices of a graph. Each vertex might receive any non-negative integer number of pebbles. A *pebbling move* consists of the following two steps:

- Remove two pebbles from a vertex.
- Place one pebble on an adjacent vertex.

We ask the following question: “Given a pebbling distribution on a graph  $G$ , and a particular vertex  $r$  of  $G$  (called the *root vertex* of  $G$ ), is it possible, via a sequence of pebbling moves, to place a pebble on  $r$ ?” The *pebbling number*  $p(G)$  of  $G$  is the minimum number of pebbles such that it is possible to pebble to any root vertex from any initial pebbling arrangement of  $p(G)$  pebbles.

1. Find the pebbling number of a cycle of length 5.
2. Find a lower bound for the pebbling number of a graph in terms of the number of vertices of the graph.
3. The *distance* between two vertices is the length of the shortest path between them. The *diameter* of a graph is the largest distance between any two vertices. Find a lower bound for the pebbling number of a graph in terms of the diameter of the graph.
4. Find three connected graphs, one for which your first lower bound is tight and your second is not, one for which your second lower bound is tight and your first is not, and one for which neither lower bound is tight.
5. Find an upper bound for the pebbling number of a connected graph. (Why is this not an interesting question for disconnected graphs?)
6. Find the pebbling number of a graph of diameter 1.
7. Again in terms of the number of vertices of the graph, what are the possibilities for the pebbling number of a graph of diameter 2?