

Math 486: Graph Theory
Fall 2008
Definitions List

- A **graph** G is a set of vertices $V(G)$, a set of edges $E(G)$, and a relation that associates with each edge two vertices called its **endpoints**.
- A **loop** is an edge whose endpoints are the same.
- **Multiple edges** are edges with the same endpoints.
- A **simple graph** is a graph with no loops or multiple edges.
- Two vertices v and w in G are **adjacent**, denoted $v \leftrightarrow w$, if there is an edge between them.
- If the vertex v is an endpoint of the edge e , then e and v are **incident**.
- The **degree** $d(v)$ of a vertex v is the number of edges incident to it.
- An **isomorphism** between two simple graphs G and H is a bijection $\varphi : V(G) \rightarrow V(H)$ such that $vw \in E(G)$ if and only if $\varphi(v)\varphi(w) \in E(H)$.
- Two graphs are **isomorphic** if there exists an isomorphism between them.
- The **complement** of a simple graph G is the graph \bar{G} with $V(G) = V(\bar{G})$, and $uv \in \bar{G}$ if and only if $uv \notin G$.
- The **path** P_n is the graph with n vertices v_1, \dots, v_n and edges $v_1v_2, \dots, v_{n-1}v_n$.
- The **cycle** C_n is the graph with n vertices v_1, \dots, v_n and edges $v_1v_2, \dots, v_{n-1}v_n, v_nv_1$.
- The **complete graph** K_n is the graph with n vertices in which every pair of vertices are adjacent.
- A graph G is **bipartite** if its vertices can be partitioned into two sets X and Y , such that every edge in $E(G)$ is from a vertex in X to a vertex in Y .
- The **complete bipartite graph** $K_{m,n}$ is the bipartite graph with parts of sizes m and n , such that every pair of vertices in different sets are adjacent.
- The **adjacency matrix** A of a graph G with n vertices is the $n \times n$ matrix with rows and columns indexed by the vertices of G , where the number in the i th row and j th column of A is the number of edges between the i th and j th vertex.
- The **incidence matrix** M of a graph G with n vertices and e edges is the $n \times e$ matrix with rows indexed by the vertices of G and columns indexed by the edges of G , where the number in the i th row and j th column of A is the number of times vertex i and edge j are incident.

- A **walk** in a graph is an alternating sequence of vertices and edges $v_1e_1v_2e_2\dots v_ke_k$, such that each term of the sequence is incident to the next.
- A **trail** is a walk with no repeated edge.
- A **path** is a walk with no repeated vertex.
- A **circuit** is a trail whose first and last vertices are the same.
- A **cycle** is a circuit with no repeated vertex other than the first and last vertex.
- The **length** of a walk, trail, path, circuit, or cycle in a graph is the number of edges in it (counting repeated edges multiple times).
- The **girth** of a graph G is the length of its shortest cycle. If G has no cycles, then its girth is infinite.
- A graph G is **connected** if, for every pair of vertices in G , there exists a path between them.
- A **component** of a graph G is a maximal connected subgraph of G .
- A **cut edge** or **cut vertex** of a graph is an edge or vertex whose deletion increases the number of components.
- A **subgraph** H of a graph G is a graph such that $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.
- An **induced subgraph** H of a graph G is a subgraph of G obtained by deleting a set of vertices.
- An **Eulerian circuit** (alternatively, **Euler tour**) of a graph G is a circuit which contains every edge of G .
- A graph is **Eulerian** if it contains an Eulerian circuit.
- A **Hamiltonian cycle** of a graph G is a cycle which contains every vertex of G .
- A graph is **Hamiltonian** if it contains a Hamiltonian cycle.
- A graph G is **regular** if every vertex of G has the same degree. If that degree is k , then G is **k -regular**.
- The **neighborhood** of a vertex v is the set of vertices adjacent to v .
- The **degree sequence** of a graph is the list of degrees of the graph, in non-increasing order.
- A **tree** is a connected graph with no cycles.
- A **leaf** of a tree is a vertex of degree 1.
- A **spanning subgraph** H of a graph G is a subgraph of G such that $V(H) = V(G)$.

- A **spanning tree** of a graph G is a spanning subgraph of G which is a tree.
- The **distance** $d(v, w)$ between two vertices v and w is the length of the shortest path between them.
- The **diameter** of a graph is the largest distance between any pair of vertices. If the graph is disconnected, then its diameter is infinite.
- A **forest** is a graph with no cycles.
- A function on the natural numbers $f(n)$ is **order** $g(n)$, denoted $O(g(n))$, if there exists positive constants c and a such that $f(n) \leq cg(n)$ for all $n \geq a$.
- The **running time** of an algorithm is the maximum number of steps it takes, as a function of the size of the input.
- A graph-related algorithm is **good** (alternatively, **fast**, **feasible**, or **polynomial-time**) if, for any graph with n vertices, the algorithm has running time $O(n^k)$ for some positive integer k .
- A **weighted graph** is a graph with numbers, called **weights**, assigned to each of its edges.
- In a weighted graph, the **distance** $d(v, w)$ between two vertices v and w is the minimum sum of the weights on the edges of a v, w -path.