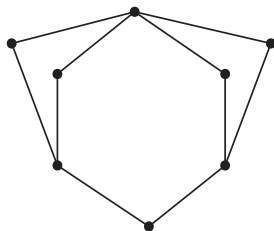


Graph Theory Assignment #9

1. The **optimal pebbling number** $o(G)$ is the smallest number of pebbles such that there exists a solvable distribution with $o(G)$ pebbles. Find the optimal pebbling number of P_n .
2. Consider a pebbling distribution on a graph with target vertex t . The pebbling step $[v, w]$ (taking two pebbles from v and placing one pebble on w) is **semi-greedy** if $\text{distance}(w, t) \leq \text{distance}(v, t)$. The graph G is **semi-greedy** if every distribution on G with $p(G)$ pebbles can be solved using only semi-greedy pebbling steps.

Show that the graph shown below is not semi-greedy.



3. The **circulant graph** $\text{Circ}(n, \{k_1, \dots, k_m\})$ is the graph with n vertices arranged in a circle, and an edge between two vertices if their distance around the circle is in the set $\{k_1, \dots, k_m\}$.
 - (a) Draw $\text{Circ}(13, \{1, 5\})$ and use it to show that $R(3, 5) = 14$.
 - (b) Draw $\text{Circ}(17, \{1, 2, 4, 8\})$ and use it to show that $R(4, 4) = 18$.
4. Prove that P_n is graceful.
5.
 - (a) Find a magic labeling of K_5 .
 - (b) Find a magic labeling of $K_{3,3}$.