

## Math 253 – Linear Algebra

Fall 2007

### Individual Problems #5, Due Wednesday, November 7

Determine whether the following statements are true or false, and prove it.

1. If  $A$  is a square matrix such that  $A^T A = I_n$ , then  $\det(A) = \pm 1$ .
2. There exists a  $4 \times 4$  matrix such that  $\det(A) = \det(4A)$ .
3. There exists a  $3 \times 3$  matrix  $A$  such that  $\text{Col}(A) = \text{Nul}(A)$ .
4. There exists a  $4 \times 4$  matrix  $A$  such that  $\text{Col}(A) = \text{Nul}(A)$ .
5.  $\{A \mid \det(A) = 1\}$  is a subspace of  $M_{2 \times 2}$ .
6. If  $\{\vec{b}_1, \dots, \vec{b}_n\}$  is a basis of  $\mathbb{R}^n$ , and  $A$  is an invertible  $n \times n$  matrix, then  $\{A\vec{b}_1, \dots, A\vec{b}_n\}$  is a basis of  $\mathbb{R}^n$ .