1. Describe the possible sets which are the ranges of linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$. For each set, give an example of a linear transformation with that range.

2. “If $T : \mathbb{R}^p \to \mathbb{R}^n$ is a linear transformation, and $U : \mathbb{R}^m \to \mathbb{R}^p$ is a linear transformation, then the composite function $T(U)$ is a linear transformation.” Prove this statement two different ways, using the two equivalent definitions of a linear transformation.

3. For a constant vector $\vec{b}$ in $\mathbb{R}^3$, define $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T(\vec{x}) = \vec{x} \times \vec{b}$, i.e., the cross product of the vectors $\vec{x}$ and $\vec{b}$.
   
   (a) Prove that $T$ is a linear transformation.
   
   (b) Is $T$ onto? Justify your answer.