

## Math 253 – Linear Algebra

Fall 2007

### Team Problems #2, Due Wednesday, September 26

1. Describe the possible sets which are the ranges of linear transformations from  $\mathbb{R}^2$  to  $\mathbb{R}^2$ . For each set, give an example of a linear transformation with that range.
2. “If  $T : \mathbb{R}^p \rightarrow \mathbb{R}^n$  is a linear transformation, and  $U : \mathbb{R}^m \rightarrow \mathbb{R}^p$  is a linear transformation, then the composite function  $T(U)$  is a linear transformation.” Prove this statement two different ways, using the two equivalent definitions of a linear transformation.
3. For a constant vector  $\vec{b}$  in  $\mathbb{R}^3$ , define  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  by  $T(\vec{x}) = \vec{x} \times \vec{b}$ , i.e., the *cross product* of the vectors  $\vec{x}$  and  $\vec{b}$ .
  - (a) Prove that  $T$  is a linear transformation.
  - (b) Is  $T$  onto? Justify your answer.