Math 253 – Linear Algebra
Fall 2007
Team Problems #3, Due Monday, October 8

Each of the following are properties of real numbers. Determine which of them are also true about matrices, and prove it.

1. For real numbers, if \( ab = 0 \), then either \( a = 0 \) or \( b = 0 \). For matrices, is it true that if \( AB \) is the zero matrix, then either \( A \) is the zero matrix or \( B \) is the zero matrix?

2. For real numbers, if \( a^2 = 1 \), then \( a = 1 \) or \( a = -1 \). For matrices, is it true that if \( A^2 = I_n \), then either \( A = I_n \) or \( A = -I_n \)?

3. For real numbers, \( a(b+c) = ab + ac \). For matrices, is it true that \( A(B+C) = AB + AC \)?

4. For real numbers, if \( ab = cb \) and \( b \neq 0 \), then \( a = c \). In the standard problems, you saw this was not true for matrices. Here’s a revised version that might be true. If \( A \) and \( B \) are two \( m \times n \) matrices such that \( A\vec{v} = B\vec{v} \) for all \( \vec{v} \in \mathbb{R}^n \), then \( A = B \).