

Math 253 – Linear Algebra

Fall 2007

Team Problems #3, Due Monday, October 8

Each of the following are properties of real numbers. Determine which of them are also true about matrices, and prove it.

1. For real numbers, if $ab = 0$, then either $a = 0$ or $b = 0$. For matrices, is it true that if AB is the zero matrix, then either A is the zero matrix or B is the zero matrix?
2. For real numbers, if $a^2 = 1$, then $a = 1$ or $a = -1$. For matrices, is it true that if $A^2 = I_n$, then either $A = I_n$ or $A = -I_n$?
3. For real numbers, $a(b+c) = ab+ac$. For matrices, is it true that $A(B+C) = AB+AC$?
4. For real numbers, if $ab = cb$ and $b \neq 0$, then $a = c$. In the standard problems, you saw this was not true for matrices. Here's a revised version that might be true. If A and B are two $m \times n$ matrices such that $A\vec{v} = B\vec{v}$ for all $\vec{v} \in \mathbb{R}^n$, then $A = B$.