

## Math 253 – Linear Algebra

Fall 2007

### Team Problems #4, Due Friday, October 26

1. Find two matrices  $A$  and  $B$  such that  $\text{Col}(A) = \text{Col}(B)$ , and  $\text{Nul}(A) = \text{Nul}(B)$ , but  $A \neq B$ .
2. (Section 4.5 #32) Prove that if  $V$  and  $W$  are isomorphic vector spaces, then  $\dim(V) = \dim(W)$ .
3. Suppose that  $H$  is a set of points in  $\mathbb{R}^n$ . Prove that the following three statements are equivalent.
  - (a)  $H$  is a subspace of  $\mathbb{R}^n$ .
  - (b)  $H = \text{Col}(A)$  for some matrix  $A$ .
  - (c)  $H = \text{Nul}(B)$  for some matrix  $B$ .