**Math 253 – Linear Algebra**
**Fall 2007**
**Team Problems #6, Due Monday, November 19**

1. Suppose $P(\lambda)$ is the characteristic polynomial of the $n \times n$ matrix $A$.

   (a) Show that $P$ has degree $n$.

   (b) Show that the coefficient of $\lambda^n$ in $P$ is 1.

   (c) The ***trace*** of a matrix $A$ is the sum of its diagonal entries. Show that the coefficient of $\lambda^{n-1}$ in $P$ is $-\operatorname{trace}(A)$.

   (d) Show that the constant term of $P$ is $(-1)^n \det(A)$.

2. Suppose that $T(\vec{x}) = A\vec{x}$ is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^n$. A subspace $V$ of $\mathbb{R}^n$ is said to be ***invariant*** under the linear transformation $T$ if whenever $\vec{x}$ is in $V$, $T(\vec{x})$ is also in $V$.

   (a) Prove that a one-dimensional subspace $V$ of $\mathbb{R}^n$ is invariant under $T$ if and only if $V$ is contained in an eigenspace of the matrix $A$.

   (b) Find an example of a matrix $A$ and a proper subspace $V$ of $\mathbb{R}^n$ such that $V$ is invariant under $T$, but $V$ is not contained in an eigenspace of $A$. (Hint: What must $T$ do to $V$ geometrically?)