

Math 253 – Linear Algebra

Fall 2007

Team Problems #8, Due Friday, December 15

1. Suppose that A is an $m \times n$ matrix. Prove that the maximum number of linearly independent rows of A is equal to the maximum number of linearly independent columns of A .
2. An $n \times n$ matrix A is called *skew-symmetric* if $A^T = -A$.
 - (a) Suppose that A is an $n \times n$ skew-symmetric matrix. If n is odd, prove that A is not invertible.
 - (b) Find a 4×4 invertible skew-symmetric matrix.
3. Suppose that V and W are vector spaces, $T : V \rightarrow W$ is a linear transformation, and S is a subspace of W . Define the ***inverse image*** $T^{-1}(S) = \{\vec{v} \in V \mid T(\vec{v}) \in S\}$.
 - (a) (Section 4.2 #36) Prove that $T^{-1}(S)$ is a subspace of V .
 - (b) Prove that $\dim(T^{-1}(S)) \geq \dim(S)$.
 - (c) What is $T^{-1}(S)$ called if $S = \{0\}$?