1. (3 points) What causes null pointer exceptions?

2. (3 points) Write a few lines of code that will cause a null pointer exception.

3. (3 points) Why is it important to keep your code simple?

4. (5 points) When is an algorithm said to be \( \Theta(f(n)) \)? Or, what is the definition of \( \Theta \)?

5. (6 points) What does it mean to trade space for time? How does dynamic programming illustrate this (give an example)?

6. (6 points) What is the definition of a binary tree?

7. (9 points) Write pseudocode for merge sort. What is its running time, in \( \Theta \) notation?
8. (8 points) What is the definition of a heap? Keep the smallest element at the root.

9. (6 points) Write pseudo-code to insert an element in a heap.

10. (5 points) What is the running time of insert (above)? Why?

11. (7 points) Write pseudo-code for heap sort.

12. (4 points) When does a heap become degenerate? What is the running time of heap sort in that case?

13. (4 points) What are the two steps in a proof by induction?
   
   i
   
   ii
14. (15 points) Write an NaryTree class for use as a game tree, which has a root (a Board), and a list of children. Include a constructor which is passed a Board, and public String toString() which returns the entire tree, with each Board properly indented. Assume Board has a String toString(String indent) method.
15. (4 points) As seen in class, if you attempt to build a complete game tree for even a tiny game like tic-tac-toe, it may not fit in memory. To solve this problem we eliminated duplicate Boards, but, the simplest method of detecting duplicated (searching the tree) didn’t work out so well. Why not?

16. (4 points) (continuing) So... if you had a method to create a Gödel number from a Board, you could use a `HashMap<Integer, NaryTree>`, to store the locations of all the unique Boards in the tree. Assuming a `GödelNumber` method exists in Board, write the line of code you would include in your constructor (above) to add the new Board to the HashMap.

17. (8 points) Write a `GödelNumber` method for Board. Assume the state of the Board is stored in an `ArrayList<Integer>` with 9 elements, each of which is either 0, 1, or 2. Treat this as a representation of a 9-digit base-3 number, and return its value in decimal.

Extra Credit (10 points) – must be written on the back of page 3! Prove by induction that $2^n > n^2$ for all $n > 5$