Abstract

Most theoretical work on the formation of IEAs has implicitly assumed that trade relationships between potential signatories do not exist. Our work extends the standard participation game model to explicitly consider the impact of trade on the size and effectiveness of an IEA in a world of short-sighted, self-interested countries (negotiators) that live up to their bargains. As the impact of trade can be positive or negative, we decompose it into three components: scale effects; smoothing effects; and leakage effects. We show that leakage leads to a larger IEA with lower individual abatement targets, while scale effects and smoothing effects tend to lead to a smaller agreement with greater individual abatement targets. Moreover, of the three effects, only leakage has the potential to induce full cooperation when it does not exist under autarky. Thus, leakage has a surprisingly positive effect on the negotiation of an IEA.
1 Introduction

Some environmental problems are global in scope; domestic activities create environmental damage that is felt by all other countries. Emitting greenhouse gases or releasing CFCs that damage the ozone layer are classic examples. In such cases a cooperative approach to the problem is essential. If instead countries regulate unilaterally, standard economic models predict that the outcome will be inefficient because regulation is a global commons game (analogous to Hardin [21]). No country will appropriately value the positive externalities created by limiting these activities; consequently, regulation will be undertaken at an inefficiently low level (if it is undertaken at all).

Negotiating an international environmental agreement (IEA) such as the Kyoto Protocol can be seen as attempt to promote efficiency by creating a framework to facilitate cooperation. However, because there is no supranational authority to design and enforce such an agreement, it must be “self-enforcing.” That is, countries must voluntarily join and comply with the terms of the treaty. Given these limitations, how effective can we expect a self-enforcing IEA to be in promoting an efficient response to global environmental problems?

The attempt to answer this question has given rise to an extensive literature\(^1\) beginning with the seminal paper by Scott Barrett [1]. Borrowing from the literature on cartel stability (see D’Aspremont, et al [15]), Barrett modeled the negotiation of an IEA as a simple noncooperative game. The structure of an agreement is determined by the best interests of the signatory nations (group rationality), and a country will join the agreement only if it in its best interests to do so (individual rationality). Using a series of numerical simulations, Barrett demonstrated that the equilibrium agreement tends to fall well short of achieving the efficient outcome. Intuitively, each country prefers to consume the benefits provided by such an agreement without incurring the costs of participating in it; that is, each nation has an incentive to free-ride that undermines the effectiveness of an IEA. Moreover, he found that other things being equal, the incentive to free-ride increases with the number of potential signatories or the severity of the environmental problem (up to a point\(^2\)), decreasing the effectiveness of the agreement.

\(^{1}\)Useful surveys of this literature include Carraro and Siniscalco [12], Wagner [31], and Ioannidis, Papandreou, and Sartzetakis [22].

\(^{2}\)If the environmental problem is severe enough, then there may be significant, even complete, participation in the agreement. However, Barrett found this to be true only when there is little difference between unilateral and cooperative regulation, and thus, is
Many subsequent papers have extended this model, exploring the limits of these pessimistic results. In particular, the impact of: i) alternate definitions of rationality; ii) heterogenous countries; iii) repeated negotiations; iv) uncertainty; v) different agreement structures; and vi) linking the negotiation of an IEA to some other international issue have all been considered.

However, there has been little systematic consideration of the impact of trade on the effectiveness of an IEA. In fact nearly all of the models in this literature explicitly or implicitly assume that there is no international trade. The purpose of this paper is to take a first step toward filling that gap. Following Barrett [3] we introduce a model in which trade can be explicitly analyzed and ask: Do the results from Barrett [1] generalize when there is trade between countries? How does trade alter participation in and effectiveness of an IEA? Using a combination of analytic and simulation techniques, we derive a number of interesting results. We show that the underlying shortcomings of an IEA indentified by Barrett [1] are still present when there is trade. However, relative to autarky the impact of trade on participation and effectiveness is ambiguous, so we decompose it into three components: scale effects; smoothing effects; and leakage effects. We show that leakage leads to a larger IEA with lower individual abatement targets, while scale effects and smoothing effects lead to a smaller agreement with greater individual abatement targets. Moreover, of the three effects, only leakage has the potential to induce full cooperation when it does not exist under autarky. Thus, leakage has a paradoxically positive effect on the negotiation of an IEA.

The rest of the paper is organized as follows. In Section 2 we present the simple model on which our analysis is built. In Section 3 we introduce some simple analytic tools to construct an interpretive framework for the simulation results discussed in Sections 4 and 5. We compare participation little need for an IEA to promote efficiency anyway.

3Eg, Chander and Tulkens [13], Tulkens [28], Ecchia and Marriotti [16] and [17]
4Eg, McGinty [25], Botteon and Carraro [7] and [8], Barrett [2]
5Eg, Barrett [4], Finus and Rundshagen [19]
6Eg Na and Shin [26], Fujitsu [20], Ulph [30], Kolstadt [24]
7Eg Bucholz, Haupt, and Peters [9], Finus, Altamarino-Cabrera, and Van-Ierland [18], Black, Levi, and De Maza [6]
8Eg Barrett [3], Carraro and Siniscalco [11]
9Barrett [3] explicitly models trade but restricts analysis to exploring the potential to strategically employ trade policy to induce participation in an IEA. Kemfert, Lise, and Tol [23] also explicitly model trade but only consider a scenario in which there are three potential signatories to an agreement, making systematic comparisons difficult.
in and effectiveness of an IEA under autarky and trade via simulation results in Section 4 and then probe for further insight by decomposing the impact of trade into smoothing, scale and leakage effects in Section 5. We conclude with a discussion of our results and questions for further investigation in Section 6.

2 The Model

Our model is a slight generalization of that used by Barrett [3] in which there are \( n \) identical firms, one in each of \( n \) identical countries. Each firm produces a single composite good \( x \) and in the process generates polluting emissions \( e \) that create global damages.

2.1 Negotiating an IEA

Negotiating an agreement is modeled as a single-shot, noncooperative game with the following four stages:

- **Stage 1:** All countries decide simultaneously (and independently) whether or not to join an agreement.

- **Stage 2:** Signatories—countries that chose to join the agreement in stage 1, we will denote them with the subscript \( s \)—collectively choose their abatement targets. Note that since countries and firms are symmetric, the agreement will assign the same abatement target to each signatory. It is the level of this target that is being chosen here.

- **Stage 3:** Nonsignatories—countries that chose not to join the agreement in stage 1, we will denote them with a subscript \( n \)—unilaterally set their abatement targets, taking signatory abatement targets as given.

- **Stage 4:** Firms choose production levels to maximize profit, given the domestic environmental regulation imposed in stages two and three.

A negotiated agreement is a subgame perfect equilibrium of this game; all actors (individuals and groups) are assumed to behave rationally both on and off the equilibrium path. Thus, we assume cooperation cannot be induced through the use incredible threats.
2.2 Payoffs

Let \( x_i \) and \( \hat{x}_i \) be the amounts of \( x \) produced and consumed in country \( i \), respectively. Both the costs of production in country \( i \), \( c(x_i, q_i) = \sigma q_i x_i + \alpha q_i \), and emissions generated by country \( i \), \( e(x_i, q_i) = (1 - q_i) x_i \) are determined by the level of production and by the abatement target \( (0 \leq q_i \leq 1) \). Consumers in each country care both about ordinary goods and services and environmental quality. Demand for goods and services in country \( i \) is captured by the inverse demand function \( p(\hat{x}_i) = b - m \hat{x}_i \). Because of the global nature of environmental problems in this model, the pollution damages in country \( i \) depend only on total emissions and not on the distribution, \( D(E) = \omega E = \omega \sum_{i=1}^{n} e_i \).

A firm’s payoff is measured by its profit:

\[
\pi_i = p(\hat{x}_i) x_i - c(x_i, q_i)
\]

Under autarky, domestic consumption must equal domestic production; so \( \hat{x}_i = x_i \) and \( p(\hat{x}_i) = p(x_i) \). When there is free trade, prices equalize in all countries and symmetry of demand implies that each country will consume an equal share of total production, so, \( \hat{x}_i = \frac{\sum_j x_j}{n} \) and \( p(\hat{x}_i) = p \left( \frac{\sum_j x_j}{n} \right) \).

A country’s payoff is measured by the welfare of its citizens. Thus, its payoff function is the sum of domestic surplus minus the loss in welfare due to environmental degradation:

\[
P_i = \pi_i + \int_{0}^{\hat{x}_i} p(z) dz - p(\hat{x}_i) \hat{x}_i - D(E)
\]

where \( \hat{x}_i = x_i \) under autarky and \( \hat{x}_i = \frac{\sum_j x_j}{n} \) under free trade as noted above.

2.3 Equilibrium

We can find the subgame perfect equilibrium of this game using backward induction, which means we must find the Nash equilibria of a sequence of nested subgames. We briefly describe each of the subgames below.

**The Firms’ Subgame:**

The smallest subgame is the production game played by firms for a given partitioning of countries (into signatories and nonsignatories) and a given distribution of abatement targets, \( \vec{q} \). The trade regime also plays a significant role in defining the form (market structure) of this game. Under
autarky, each firm acts as a domestic monopolist. By contrast, when there is trade the firms compete with each other in Cournot oligopoly.

Regardless of the market structure, each firm will choose $x_i$ to maximize profit, assuming that the production levels of all other firms are fixed and that markets clear. A Nash equilibrium of this game is a simultaneous solution to $n$ maximization problems of the form:

$$\max_{x_i} \quad p(\hat{x}_i) x_i - c_i(x_i, q_i)$$
$$\text{s. t.} \quad x_i \geq 0$$

(1)

As noted above, the form of $\hat{x}_i$ depends on the trade regime, and in turn determines the market structure.

**The Nonsignatories’ Subgame:**

Given a partitioning of countries and a signatory abatement target, $q_s$, nonsignatories play a subgame in which each chooses an abatement target to maximize its payoff, assuming that all the abatement targets of all other countries are fixed and anticipating the solution of the firms’ subgame. Assuming that there are $k$ signatories, a Nash equilibrium of this subgame is simultaneous solution to $n - k$ maximization problems of the form:

$$\max_{q_i} \quad \pi_i + \int_0^{\hat{x}_i} p(r) dr - p(\hat{x}_i) \hat{x}_i - \omega E$$
$$\text{s. t.} \quad x_j = x_j(q_j, \vec{q}_{-j}), \forall j, \hat{x}_j = \hat{x}_j(q_j, \vec{q}_{-j}), \forall j$$
$$0 \leq q_i \leq 1$$

(2)

By symmetry, the solution to this problem will be the same for each nonsignatory, $q_i^* = q_j^* = q_n$. In what follows we will restrict attention to that part of the parameter space in which $q_n = 0$.

**The Signatories’ Subgame:**

In the penultimate subgame, a set group of $k$ signatories cooperate to choose abatement targets that maximize their collective payoffs, anticipating the outcomes of the firms’ and nonsignatories’ subgames. As noted above, symmetry implies that each signatory will be assigned the same target. Thus, the equilibrium of this subgame is the solution to following maximization
problem:

$$\max_{q_s} \quad k \left( \pi_i + \int_0^{\hat{x}_i} p(r) \ dr - p(\hat{x}_i) \hat{x}_i - \omega E \right)$$

s. t. \hspace{1cm} x_j = x_j(q_j, \vec{q}_{-j}), \forall j
\hspace{1cm} \hat{x}_i = \hat{x}_j(q_i, \vec{q}_{-j}), \forall j
\hspace{1cm} q_j = 0, \forall j \text{ that is a nonsignatory}
\hspace{1cm} q_i = q_s, \forall i \text{ that is a signatory}
\hspace{1cm} 0 \leq q_s \leq 1$$

We call the solution to this problem, $q_s(k)$, the *intensity function* because it indicates “how hard” a group of $k$ signatories will try to abatement emissions.

**The Participation Subgame**

A subgame perfect equilibrium, then, is a Nash equilibrium of ultimate subgame in which countries simultaneously decide whether or not to join the agreement, anticipating the outcome of the three subsequent subgames. That is, it is a partitioning of countries into signatories and nonsignatories from which no country has incentive to deviate.

More formally, let $P_s(k)$ and $P_n(k)$ be the payoffs to being a signatory or a nonsignatory to an agreement with $k$ total signatories, respectively. Then the *incentive function* $I(k) = P_s(k+1) - P_n(k)$ measures the incentive a nonsignatory has to join an agreement that already has $k$ signatories. If $I(k) \geq 0$, then nonsignatory would be better off joining the agreement rather than remaining a nonsignatory, and vice versa. So a partitioning of countries into $k$ signatories and $n-k$ non-signatories, is a subgame perfect equilibrium if and only if:

$$I(k-1) \geq 0$$  \hspace{1cm} (4a)

$$I(k) < 0$$ \hspace{1cm} (4b)

Inequality (4a) insures that no signatory has an incentive to leave the agreement and become a non-signatory to an agreement of size $k-1$. Likewise, inequality (4b) insures that no non-signatory has an incentive to join agreement, becoming a signatory to an agreement of size $k+1$. 

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2.4 Parameter Restrictions

Throughout the paper we will restrict the parameter space in the following way:

\[
0 \leq \sigma \leq \frac{b}{n-1} \quad 0 \leq \alpha \leq \frac{3b(b-\sigma)^2}{m} \quad \frac{(3\sigma b m (b-\sigma)+4\alpha m^2 b)}{(2n m b (b-\sigma) \lambda-3n \sigma^2 (b-\sigma)-8n m \sigma \alpha)} \leq \omega \leq \bar{\omega} \quad n \geq 6
\]

These assumptions are made primarily for ease of exposition. They guarantee interior solutions to the firms’ subgame and restrict our attention to situations in which there is a significant difference between globally efficient emission reductions and those undertaken when each country acts unilaterally. In particular, the value of \( \bar{\omega} \) is determined so that nonsignatories choose to not abate their emissions at all in each of the scenarios we investigate, regardless of the abatement targets chosen by signatories.

3 Effectiveness, Participation, and Intensity

Ultimately, we are interested in the effectiveness of an IEA under trade versus autarky. However, before before we move on to this comparative analysis, it is worth developing some simple analytic tools to expose the strong relationships between the effectiveness of an agreement, the number of signatories (participation), and the intensity function, \( q_s(k) \).

By the effectiveness of an IEA, we mean the answer the question: How close does it come to achieving the efficient reduction in emissions? Formally, we will measure the effectiveness of an IEA as the emissions reduction that it achieves as a percentage of the desired (efficient) reduction:

\[
\frac{E_U - E^*}{E_U - E_{FC}}
\]

where \( E_U, E_{FC}, \) and \( E^* \) are global emission when all countries act unilaterally, cooperatively, and under a negotiated IEA, respectively. Note that the emissions reduction achieved by an agreement \( E^* \) (and thus its effectiveness) is determined by the number of signatories (participation) and the abatement targets they undertake (intensity).

Likewise, there is a strong relationship between the incentive to join an agreement and the intensity function, captured in the following proposition.
Proposition 1 The incentive function can be decomposed into the difference between the benefit of joining an agreement of size $k$, $\text{Ben}(k)$, and the cost of joining an agreement of size $k$, $\text{Cost}(k)$, where:

$$\text{Ben}(k) \approx \frac{\partial P_i}{\partial q_s} (q_s(k+1) - q_s(k))$$  \hspace{1cm} (7)$$

$$\text{Cost}(k) \approx \frac{\partial P_i}{\partial q_i} k (q_s(k+1) - q_n)$$  \hspace{1cm} (8)$$

Proof: The proof of this proposition is left to appendix section (A.1).

Intuitively, equation (7) reveals that the benefit of joining an agreement is directly related to the change in abatement that is induced by doing so. In particular, if the current signatories do not change their behavior at all, then there is no benefit in joining. By contrast, equation (8) reveals that the cost of joining an agreement is related to the increase in abatement that the nonsignatory must undertake in order to join the agreement. If there is no difference between the abatement undertaken by a signatory and a nonsignatory, there is no cost to joining the agreement. Thus, the benefit of joining an agreement is related to the slope of the intensity function, while the cost of the agreement is related to the height of the intensity function.

Figure (3) captures some of the important elements of the relationship between intensity and incentive functions. Consider the s-shape of the intensity function first. For small groups of signatories, the collectively optimal abatement level will be the same as that of nonsignatories, which accounts for the initial flat section of this curve. Put differently, there is a kind of natural minimum ratification clause (a la Black, et al [6]). Unless an agreement attracts a minimum number of signatories, it will have no force; it will not require anything of signatories that they would not undertake absent the agreement. The minimum ratification level is “natural” because it arises without being explicitly introduced or negotiated. It will depend on the specific parameter values, but will always be at least two countries$^{10}$. Once the minimum ratification level is reached, however, there is some collective payoff to abating more than nonsignatories. At the margin, these benefits increase with the number of signatories$^{11}$; so, we would expect the optimal abatement to increase with $k$ also, until an agreement size is (possibly)

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$^{10}$To see why this is true, consider the case of $k = 1$ (one signatory). Since $q_n$ does not depend on $q_s(k)$ ($q_n = 0$), the signatory’s problem when $k = 1$ is identical to the nonsignatory’s problem and will have the same solution.

$^{11}$Signatories’ marginal benefit of abatement $= k \frac{\partial e_s}{\partial q_s}$. 

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reached for which signatories are abating all of their emissions and \( q_s(k) = 1 \). Signatories to larger agreements are not able to increase abatement beyond this point, which accounts for the final flat section of the curve.

Figure 1: Relationship between the incentive and intensity functions.

As suggested by equations (7) and (8), the shape of the intensity function dictates the shape of the incentive function. As seen in Figure (3), initially there is neither benefit nor cost to joining an agreement because signatories abate no more than nonsignatories. However, there must be a positive incentive to join and be the first signatory to induce an abatement increase. As the intensity function increases, the cost of joining grows. So, unless signatories are willing and able to increase their abatement enough to offset the greater cost of joining, the incentive to join an agreement will decrease with the agreement size. Once signatories have “maxed out” \( (q_s(k) = 1) \), there is a negative incentive to join. The first agreement for which there is a negative incentive to join, will be the equilibrium agreement \( (k* \) in Figure (3).

These simple analytics yield three important insights:

1. An IEA will always have some positive impact. As noted above, at least two countries will participate in an IEA, and signatories to an agreement beyond the minimum ratification level are guaranteed to be abating more than nonsignatories.

2. If countries “try too hard, too soon,” then an IEA will not have com-
plete participation and will not yield the efficient reduction level. Non-signatories have an incentive to join an agreement only if current signatories are able to reward them by increasing their abatement levels. So, if signatories to a small agreements reach their maximal abatement, then there will be no incentive for the remaining nonsignatories to join.

$$q_s(k)$$

$$q_n$$

$I(k)$

# sigs

$\# \text{sigs}$

Figure 2: Increasing intensity tends to decrease participation.

3. **Anything that tends to increase the intensity of an agreement, will tend to decrease participation.** Increasing intensity, shifts the intensity function left. Other things equal, this raises the cost of joining agreements of all sizes while decreasing the benefit of joining larger agreements, which decreases the size of the equilibrium agreement as illustrated in Figure (2). It is worth noting that this observation constitutes a partial comparative statics result. A change that affects intensity will typically affect the incentive function directly as well. This direct effect and the indirect effect (through the intensity function) may be opposing. In such cases, the ultimate impact on incentives and participation will depend on which effect dominates. However, this partial result is useful in helping interpret seemingly paradoxical simulation results. For example, as the severity of the environmental problem increases, we would signatories to increase their abatement targets, shifting the intensity function left. The increased intensity makes it more costly to join the agreement, and consequently participation in
an IEA decreases as the severity of the problem increases as seen in Barrett [1].

We now turn our attention to comparing IEAs under autarky and trade.

4 Autarky vs Trade

Is an IEA more effective under free trade than under autarky? We address this question using numerical simulation. First we solve the Nash equilibria of the three subgames under autarky and trade, respectively. These equilibria are characterized by closed-form solutions in appendix sections (A.2) and (A.3). Back-substituting these values, we are able to calculate incentive functions under autarky and free trade, respectively. Numerically solving for the roots of the incentive function, we determine the size of the equilibrium agreement for a variety of parameters in each case. Once we have identified the size of the equilibrium agreement, we are able to calculate its effectiveness by making the appropriate substitutions.

4.1 Simulation Results

Figure 3 compares the equilibrium outcomes under each trade regime for three different cases: 10 potential signatories \((n = 10)\); 50 potential signatories \((n = 50)\); and 100 potential signatories \((n = 100)\). In each case, the number of signatories to an IEA and the effectiveness of the agreement are represented for \(b = 1\), \(m = 1\), \(\sigma = .01\) \(\alpha = 0\), and numerous values of \(\omega \in [.0015, .011]\).

This figure illustrates several important points:

1. **Under both trade and autarky, as the severity of the environmental problem (indexed by \(\omega\)) or the number of potential signatories increases, there is less meaningful participation in an IEA and the agreement is less effective.** Note that although participation in the agreement is monotonically decreasing over the range of calculated values, the effectiveness of the agreement is not; it is decreasing in overall trend, but increasing over short intervals. The reason for this non-monotonicity is straightforward. Because the number of signatories to an equilibrium agreement will always be an integer, there will be ranges of values of \(\omega\) for which participation in the agreement will be the same. Since intensity increases with the severity of the environmental problem, the effectiveness of the agreement will actually
Figure 3: Comparing autarky and free trade.
increase with $\omega$ over this range. However, as $\omega$ increases outside this range, participation drops, which decreases the effectiveness of the agreement. Thus, for small changes, an IEA may be more effective when the environmental problem is more severe. But over a wide range of changes, the effectiveness of the agreement tends to decrease with the severity of the environmental problem. This is consistent with the results from previous literature and suggests that trade (as captured in this model) does not fundamentally alter the incentive to free-ride indentified by Barrett [1].

2. Relative to autarky, an agreement may achieve more or less participation when there is trade. Likewise, it may achieve a larger or smaller percentage of the desired reduction in emissions.

3. For larger groups of potential signatories, the agreement tends to have greater participation under trade than autarky but achieve a smaller percentage of the desired emissions reduction.

In order to better understand what is driving these results (particularly the last two), we will decompose the impact of trade into three principal components and examine each in isolation. We will return to further explain these results in Section 5.4.

5 Decomposing the Impact of Trade

The principal differences between autarky and trade in our model show up in the firms’ subgame and are captured in the following proposition.

**Proposition 2** In the Nash equilibrium of the Firms’ subgame, production, consumption and price may be characterized as follows:

Under autarky:

- $x_A^i(q_i) = \frac{b - \sigma q_i}{2m}$
- $\hat{x}_A^i(q) = x_A^i$
- $p_A^i = p(x_A^i)$

Under trade:

- $x_T^i(q_i) = \theta (x_A^i + \lambda (q - q_i))$
- $\hat{x}_T^i(q) = \frac{\sum x_T^j}{n}$
- $p_T^i = p \left( \frac{\sum x_T^j}{n} \right)$

where $q = \sum q_i/n$, $\theta = \frac{2n}{n+1}$, and $\lambda = \frac{n \sigma}{2m}$.


The proof of this proposition can be found in appendix sections (A.2) and (A.3).

Proposition 2 allows us to see that the impact of trade in our model can be decomposed into three components:

- **The Smoothing effect**: Under autarky, each country is a market unto itself. So, price and consumption may differ from country to country, depending, among other things, on domestic environmental regulations. However, when there is international trade, it is as if all countries are part of a single market. So, the price of the composite good will be the same in all countries, and because demand is symmetric, consumption will also be identical across countries.

- **The Scale Effect**: International trade leads to an increase in global production and consumption of the composite good (captured by $\theta$ in Proposition (2)). In our model this arises because, under autarky each firm acts as a monopolist and thus restricts production in order maximize profit. Thus, trade increases production because it increases competition. More generally, we might expect trade to increase the scale of production and consumption because of the role trade plays in creating economic growth.

- **The Leakage Effect**: Leakage refers to the fact that in the presence of trade, production of a polluting good will shift (“leak” out) away from signatories toward nonsignatories (captured by $\phi$ in Proposition (2)). Signatories increase their levels of environmental protection, thereby increasing the cost of producing a polluting good. Thus, signatories have decreased their comparative advantage in the polluting goods and a greater share of global production will be undertaken by nonsignatories (see Baumol and Oates [5], Chapter 16).

In what follows, we analyze each effect in isolation. Note that if we allow firms to determine production (and consumption) levels optimally as part of the equilibrium outcome, then all three of these effects (the smoothing effect, scale effect, and leakage effect) will be present in the result. Thus, in order to examine each of the effects in isolation, we will determine production and consumption levels by assumption rather than solving for them. However, abatement targets and participation decisions will be chosen endogenously as before.
5.1 Impact of the Smoothing Effect

Recall that smoothing refers to the fact that under free trade price and consumption levels will be the same in each country. In order to isolate smoothing from scale and leakage effects, we make the following assumptions:

\[ x^M_i(q) = x^A_i(q) = \frac{1 - \sigma q_i}{2} \; \text{; and} \]
\[ \hat{x}^M_i = \frac{\sum_j x^A_j(q)}{n} = \frac{1 - \sigma \frac{\sum_j q_j}{n}}{2} \; \text{(10)} \]

Here equation (9) eliminates leakage and scale effects, while equation (10) produces the smoothing effect.

How does price and consumption equalization impact the formation of an IEA? Results of our analysis are presented in Proposition 3 and Figure 4 below. Figure 4 illustrates the results of numerical simulations for the case of 10 potential signatories, using the same parameter values as were used in section 4. The proof of Proposition 3 is left to appendix section (A.4).

**Proposition 3** Smoothing increases the intensity of an agreement of any size: \( q^M_{e}(k) \geq q^A_{e}(k) \).

![Simulation results for autarky and trade with isolated smoothing effect.](figure)

Under autarky, the marginal cost of abatement arises in part from reduced consumption. Smoothing reduces these costs because the same change in domestic abatement leads to a smaller drop in domestic consumption, part of the reduction is in domestic production is born by other countries.
Consequently, it is worthwhile for any group of signatories to abate more than they would have done under autarky. As explained in section 3, other things being equal, we might expect the increase in abatement to increase the cost of joining an agreement and, thus, lower equilibrium participation. However, as Figure 4 shows, in our model the change in intensity induced by smoothing is so small that participation levels and effectiveness are virtually unchanged due to the smoothing effect. Thus, smoothing contributes little toward explaining the impact of trade as describe in Section 4.

5.2 Impact of the Scale Effect

Recall that the scale effect refers to the fact that under free trade production and consumption levels are scaled up. In order to isolate this effect, we make the following assumptions:

\[ x_i^S(\bar{q}) = \theta x_i^A(\bar{q}) \; \text{and} \]

\[ \dot{x}_i^S(\bar{q}) = x_i^S(\bar{q}) \]

Here equation (11) eliminates leakage while producing scale effects, and equation (12) eliminates the smoothing effect. The parameter \( \theta \) can be thought of as an index of the scaling effect; for larger values of \( \theta \) there is a greater scale effect.

We compare IEAs under autarky and trade with the scale effect isolated. The results of our analysis are presented in Proposition 4 and Figure 5 below. Again, Figure 5 illustrates the results of numerical simulations for the case of 10 potential signatories, using the same parameter values as were used in Section 4 and a scale factor \( \theta = \frac{2n}{n+1} \). The proof of Proposition 4 is left to appendix section (A.5).

**Proposition 4** For \( \theta \geq \frac{b+4\sigma}{b+\sigma} \), the scale effect increases the intensity of an agreement of any size and decreases participation in the equilibrium agreement:\(^\text{12}\)

\[ q_s^S(k) \geq q_s^A(k) \]

\[ k^S \leq k^A \]

The intuition for these results is straightforward. Since emissions are a proportional to output, the marginal benefit of abatement scales up with production. By contrast, even though reducing emissions by 1% more requires reducing more total emission, the marginal cost of abatement does

\(^\text{12}\)Note \( \frac{b+4\sigma}{b+\sigma} \leq \frac{2n}{n+1} \)
Figure 5: Simulation results for autarky and trade with isolated scale effect.

not scale up with output. This is true because the level effect leads to greater consumption and lower prices, and thus reduced consumption and production (part of the cost of abatement) have less value at the margin. Consequently, it is worthwhile for any group of signatories to abate more than they would have under autarky. As noted, in Section 3, the increased intensity makes it more costly to join the agreement, and consequently, equilibrium participation is lower because of the scale effect.

The implications for effectiveness are less clear cut. Because intensity and participation are changing in opposite directions, an agreement may end up being more or less effective. This point is illustrated Figure 5. However, it is worth noting that, at least in this case \( n = 10, \theta = \frac{2n}{n+1} \), the participation change dominates, making the IEA less effective for most of the simulation runs. Moreover, although scaling up production may make an agreement more effective, there are limits to how much improvement can take place because participation is always reduced. So in particular, the scale effect cannot induce the efficient outcome. An IEA will yield the efficient outcome under trade with isolated scale effect only if it yields the efficient outcome under autarky\(^{13}\).

5.3 Impact of the Leakage Effect

Recall that leakage refers to a change in the pattern of production that occurs under trade because of differential domestic abatement targets. In

\(^{13}\)This true because an IEA will yield the efficient outcome only if all potential signatories participate.
order to isolate this effect, we make the following assumptions:

\[ x^L_i(\bar{q}) = x^A_i(\bar{q}) + \lambda(\bar{q} - q_i); \quad \text{and} \]

\[ \hat{x}^L_i(\bar{q}) = x^L_i(\bar{q}) \]

Here equation (13) eliminates scale effects while producing leakage, and equation (14) eliminates the smoothing effect. The parameter \( \lambda \) can be thought of an index of the leakage effect; for larger values of \( \lambda \) there is a greater degree of leakage.

As before we compare IEAs under autarky and trade with the leakage effect isolated, reporting our results in Proposition 5 and Figure 6 below. Again, Figure 6 illustrates the results of numerical simulations for the case of 10 potential signatories, using the same parameter values as were used in section 4 and a leakage factor \( \lambda = \frac{n \sigma}{2} \). The proof of Proposition 5 is left to appendix section (A.6).

**Proposition 5** For \( \lambda \geq \frac{\sigma (8m \alpha + 3 \sigma (b - \sigma))}{b_m (b - m)} \), the leakage effect decreases the intensity of an agreement of any size and increases participation in the equilibrium agreement\(^{14} \):

\[ q^L_s(k) \leq q^A_s(k) \quad k^{\ast L} \geq k^{\ast A} \]

**Figure 6:** Simulation results for autarky and trade with isolated leakage effect.

The intuition for these results is relatively straightforward. Leakage undermines the effectiveness of domestic abatement efforts. It increases the

\(^{14}\text{Note} \quad \frac{\sigma (8m \alpha + 3 \sigma (b - \sigma))}{b_m (b - m)} \leq \frac{n \sigma}{2m} \)
marginal cost of abatement, because competition from other firms means that the same increase in domestic abatement efforts leads to a greater decrease in production. Likewise, it reduces the value of domestic abatement because some portion of the domestic emissions reduction is offset by increased emissions elsewhere. Consequently, it is not worthwhile for any group of signatories to abate as much as they would have under autarky. As noted, in section 3, the decreased intensity increases the “natural” minimum ratification level and decreases the cost of joining an agreement, consequently, leakage increases equilibrium participation.

Once again, the implications for effectiveness are not clear cut, but the potential for improvement is not as limited as was the case for the scale effect. Intensity and participation are changing in opposite directions, so the IEA may be more or less effective, depending on which of these changes dominates. Both outcomes can be seen in Figure 6, although for this scenario \( n = 10, \lambda = \frac{n \sigma}{2} \) it is more often the case that the participation change dominates, making the agreement more effective. Finally, the potential for improvement is not limited as it was for the scale effect. As noted previously, leakage makes it credible for larger groups of signatories to do nothing more than nonsignatories, that is, it increases the natural minimum ratification level. So, if leakage is severe enough, it may be credible threaten that if participation is not complete, the IEA will have no force. Thus, leakage can induce the efficient outcome. The impact of differing degrees of leakage are illustrated in Figure 7.

![Simulation results for different leakage parameters.](image)
5.4 The Impact of Trade Revisited

In summary, we have shown that the impact of smoothing is negligible and that the scale and leakage effects are opposing. Scale effects tend to increase intensity and decrease participation in an IEA, while leakage effects tend to decrease intensity and increase participation in a IEA. Thus, the impact of trade depends on which of these two effects dominates. Either of these effects can lead to an agreement that is more effective or less effective than would be the case under autarky.

Returning to Figure 3, we are now able to add some additional intuition for the observations offered in Section 4. First, careful observation shows that in all cases where the IEA is more effective under trade than under autarky, the IEA also has more participation under trade than autarky. Based on our decomposition analysis, this suggests that these positive results are being driven primarily by the leakage effect.

Second, as the number of potential signatories increases, the leakage effect becomes relatively more important\(^{15}\). So, for larger the values of \(n\), we would expect that would more often lead to agreements in which there is greater participation but in which signatories might abate less than under autarky. This precisely what we see in participation graphs in Figure 3. However, although relatively less important, the scale effect is still significant \((\theta \approx 2)\). Since leakage and scale effects work in opposition to one another, the increase in participation achieved by trade is relatively small. In addition, the scale effect dramatically increases the emissions of nonsignatories. So, despite having greater participation, the agreement tends to be less effective under trade than under autarky.

6 Discussion

In this paper we introduced a simple model in which international trade is explicitly considered so that we might begin to analyze the impact of trade on the negotiation of a self-enforcing international environmental agreement. Important insights gained from our analysis can be summarized as follows:

- The underlying shortcomings of an IEA identified by Barrett \([1]\) are still present when there is trade. Typically the IEA does achieve full participation and thus does not achieve the efficient outcome. Other things being equal, an IEA has less meaningful participation and will

\(^{15}\)This is true because \(\theta = \frac{2\pi}{\pi+1}\) approaches 2 asymptotically, but \(\phi = \frac{n\sigma(\sigma_q-x)}{2}\) increases linearly.
be less effective as the environmental problem becomes more severe or as the number of signatories increases.

• The impact of trade can be decomposed into smoothing, scale, and leakage effects. Although the impact of smoothing is negligible, both leakage and scale effects play important, though opposing, roles in determining the impact of trade. Leakage decreases the abatement targets of an group of signatories, but leads to a larger IEA. The scale effect increases the abatement targets of any group of signatories, but leads to a smaller agreement.

• Consequently, trade may lead to an IEA that achieves more participation (if the leakage effect dominates) or less participation (if the scale effect dominates) than would be true under autarky.

• Likewise, trade may lead to an IEA that is more effective or less effective than would be true under autarky. Theoretically improvement could be driven by the scale effect (less participation with higher targets) or by leakage effects (increased participation). In our simulations, all the improvements were driven by leakage.

• Finally, only leakage has the potential to induce full cooperation. If leakage is severe enough, then the threat of no cooperative action without complete participation becomes credible.

Of all these results, the positive impact of leakage is perhaps the most surprising and deserves further comment. Leakage has been the focus of much consternation in the policy arena and the literature on trade and the environment because: i) it makes the abatement efforts of any group of countries less effective; and ii) it increases the cost of that effort (see, for example, Weyant, ed. [32]). The fear that leakage will cause regulation to be less effective and more costly led the United States Senate to draft Senate Resolution 98, which states that the Senate would not vote to ratify any agreement that committed the United States to emissions reductions and did not do the same for developing countries. Our analysis does not contradict these observations but casts them in different light. As noted in Section 3, IEAs fall short of efficiency because signatories “try too hard, too soon.” They behave in this way because threats to do otherwise are viewed as incredible; abating less would not be optimal for the signatories themselves. Leakage makes it credible to abate less and, thus, operates like a commitment device in a bargaining game. It allows signatories to behave more patiently, and thus improves the efficiency of the outcome.
6.1 Policy Implications

Although our model is more descriptive than prescriptive, several simple policy insights emerge from our analysis. First of all, as a policy measure IEAs often imperfect, but still useful. We showed in Section 3 that an IEA will always improve upon the fully noncooperative outcome. However, IEAs are often susceptible to free-rider problems problems that prevent them from achieving the efficient outcome. In order to achieve efficiency, IEAs will often need to be augmented with some additional incentive mechanism. One possibility that has been discussed in the literature is to link the negotiation of an IEA with the negotiation of some other international policy issue.

We have shown that IEAs can be more effective under trade than under autarky. Although more work needs to be done to characterize the conditions under which this is true, it is clear that coordination between trade and environmental policies will be useful. In essence, we agree with the OECD in saying:

The fuller integration of environmental and trade policies should aid in accentuating the positive environmental effects of trade and trade liberalisation and mitigating any negative effects.

(OECD [27], p. 17)

In particular, as suggested in Barrett [3], strategically linking trade policy and participation in an IEA may lead to a much more effective IEA.

Finally, in light of this quote, it is clear that careful thought must given to identifying the “negative” environmental effects of trade that should be mitigated. By most definitions, leakage would be considered a negative effect of trade, thus, it might seem to be prudent to link environmental and trade legislation in a way that minimizes the potential for leakage. However, our work suggests that efforts to “plug the leaks” might actually undermine efforts to promote cooperation in regulating international environmental problems.

6.2 Limitations and Future Research

We conclude with two thoughts about directions for future research. In this paper we intentionally built and analyzed a very simple model. While this yielded some useful and interesting insights, it also excluded some potentially important links between trade and the environment. In particular, it would be useful to develop a richer framework in which it would be possible to explore the impact of income, composition, and technique effects (a la Copeland and Taylor [14]) in addition to smoothing, scale and leakage.
Finally, given the importance of leakage in determining the effectiveness of an IEA, it would be useful to characterize the conditions that determine scope of the leakage effect. Under what circumstances will leakage be the dominant effect of trade? Under what conditions will leakage be more or less severe?

A Appendix

A.1 Proof of Proposition 1

First we introduce some additional notation. Let $P_i(q_i, k, \tilde{q}, \hat{q})$ be the payoff to country $i$ when country $i$ sets abatement target $q_i$, $k$ other countries\footnote{By symmetry it does not matter which ones.} set abatement target $\tilde{q}$, and $n - k - 1$ other countries set abatement target $\hat{q}$. Thus, $P_s(k) \equiv P_i(q_s(k), k - 1, q_s(k), q_n)$ is the payoff to a signatory to an agreement of size $k$; and $P_n(k) \equiv P_i(q_n, k, q_s(k), q_n)$ be the payoff to a nonsignatory to an agreement of size $k$.

From this we get:

$$I(k) = P_s(k + 1) - P_n(k)$$

$$= P_i(q_s(k + 1), k, q_s(k + 1), q_n) - P_i(q_n, k, q_s(k), q_n)$$

$$= (P_i(q_s(k + 1), k, q_s(k + 1), q_n) - P_i(q_s(k + 1), k, q_s(k), q_n))$$

$$- (P_i(q_n, k, q_s(k), q_n) - P_i(q_s(k + 1), k, q_s(k), q_n))$$

$$= Ben(k) - Cost(k)$$

The rest of the proposition follows from Taylor’s theorem.

$$Ben(k) = P_i(q_s(k + 1), k, q_s(k + 1), q_n) - P_i(q_s(k), k, q_s(k), q_n)$$

$$\approx \sum_{j=1}^{k} \frac{\partial P_i(q_s(k + 1), k, q_s(k + 1), q_n)}{\partial q_j} (q_s(k + 1) - q_s(k))$$

by Taylor’s approximation

$$= \frac{\partial P_i(q_s(k + 1), k, q_s(k + 1), q_n)}{\partial q_i} k (q_s(k + 1) - q_s(k))$$

by symmetry.

16By symmetry it does not matter which ones.
and
\[
\text{Cost}(k) = P_i(q_n, k, q_s(k), q_n) - P_i(q_s(k + 1), k, q_s(k), q_n)
\approx - \frac{\partial P_i(q_s(k + 1), k, q_s(k), q_n)}{\partial q_i} (q_s(k + 1) - q_n)
\] (16)
by Taylor’s approximation.

A.2 Autarky

A.2.1 Firm’s Subgame

Since there is no trade each firm acts as a monopolist in its home country. The conditions for an interior solution to the firm’s problem are are:

\[\begin{align*}
\text{FOC:} & \quad b - 2m x_i - \sigma q_i = 0 \\
\text{SOC:} & \quad -2m < 0
\end{align*}\]

The SOC is always satisfied; so the FOC is necessary and sufficient. Solving it for \(x_i\), we get:
\[x_i^A(q_i) = \frac{b - \sigma q_i}{2m}\] as long as \(\sigma \leq b\).

A.2.2 Nonsignatory’s Subgame

Nonsignatories simultaneously and unilaterally choose an abatement target to maximize their respective payoffs, anticipating the subsequent equilibrium in the goods and services market. The conditions for an interior solution are:

\[\begin{align*}
\text{FOC:} & \quad -\frac{1}{4m} (2\omega (b + \sigma) - 4m \alpha - 3b \sigma - \sigma (4\omega - 3\alpha)) = 0 \\
\text{SOC:} & \quad -\sigma \frac{2\omega (1+\sigma) - 4\omega - 3\alpha}{4m (4\omega - 3\alpha)} \leq 0
\end{align*}\]

For each of the \(n-k\) nonsignatories. The second order conditions are satisfied \(\omega \geq \frac{3\sigma}{4}\).

Case 1: If \(\omega < \frac{3\sigma}{4}\), then: \(P_i|_{q_i=0} - P_i|_{q_i=1} = \alpha + \frac{6\sigma^2 - 4\omega}{8} > 0\). So \(q_n^A = 0\).

Case 2: If \(\omega \geq \frac{3\sigma}{4}\), then by solving the FOC, we find:
\[q_n^A = \begin{cases} 0 & \text{if } \omega \leq \frac{4\alpha + 3b\sigma}{2(b+\sigma)} = \omega^A, \\ \frac{2\omega (1+\sigma) - 4\omega - 3\alpha}{\sigma (4\omega - 3\alpha)} & \text{if } \omega \in [\omega^A, \bar{\omega}^A] \\ 1 & \text{if } \omega \geq \frac{4\alpha + 3\sigma (1-\sigma)}{2(1-\sigma)} = \bar{\omega}^A \end{cases}\]

Note that \(\frac{4\alpha + 3\sigma}{2(1+\sigma)} \geq \frac{3\sigma}{4}\).
A.2.3 Signatory’s Subgame

Because \( x_i^A \) depends only on \( q_i \), the signatories problem is structurally the same as that of a nonsignatory with \( k \omega \) substituted for \( \omega \). Thus, the solution to this problem is:

\[
q^A_s = \begin{cases} 
0 & \text{if } k \leq \frac{4m_\alpha+3b_\sigma}{2k_\omega(b+\sigma)} \leq k^A_0, \\
\frac{2k_\omega(b+\sigma)-4m_\alpha-3b_\sigma}{\sigma(4k_\omega-3\sigma)} & \text{if } k \omega \in [k^A_0, k^A_1] \\
1 & \text{if } k \geq \frac{4m_\alpha+3b_\sigma-3\sigma^2}{2k_\omega(1-\sigma)} \leq k^A_1 
\end{cases}
\]

A.3 Free Trade

A.3.1 Firms’ Subgame

Under free trade firms compete in an oligopoly. The equilibrium is Nash/Cournot defined by the following \( n \) conditions:

\[
\begin{align*}
\text{FOC: } & b - m \frac{\sum_j x_j}{n} - m \frac{x_i}{n} - \sigma q_i = 0 \\
\text{SOC: } & -2m < 0
\end{align*}
\]

The SOC is always satisfied; so the FOC is necessary and sufficient. Solving it for all the \( x_i \)’s simultaneously, we find: \( x^T_i(\bar{q}) = \frac{2n}{n+1} \left( \frac{b - \sigma q_i}{2m} + \frac{n \sigma}{2m} (\bar{q} - q_i) \right) \)

where \( \bar{q} = \frac{\sum_j q_j}{n} \) and \( \sigma \leq \frac{1}{n} \) to guarantee an interior solution. Under free trade consumption is equal across countries: \( x^T_i(\bar{q}) = \frac{\sum_j x^T_j(\bar{q})}{n} \).

A.3.2 Nonsignatories’ Subgame

Nonsignatories simultaneously and unilaterally choose an abatement target to maximize their respective payoffs, anticipating the subsequent equilibrium in the goods and services market. The subgame equilibrium is defined by the following conditions, which are identical for all \( n - k \) nonsignatories:

\[
\begin{align*}
\text{FOC: } & \frac{1}{m(n+1)^2} \left( n(n+1) \omega (b+\sigma) - (n+1)^2 m \alpha - n(2n+1) b \sigma + \right. \\
& \sigma \left( (2n^3+1) \sigma - 2n^2(n+1) \omega \right) q_i - \\
& \left. \sigma \left( (2n^2-1) \sigma - 2n(n+1) \omega \right) \sum_{j \neq i} q_j \right) = 0 \\
\text{SOC: } & \frac{\sigma}{(n+1)^2} \left( (2n^3+1) \sigma - 2n^2(n+1) \omega \right) \leq 0
\end{align*}
\]

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The second order conditions are satisfied iff \( \omega \geq \frac{(2n^3+1)\sigma}{2n^2(n+1)} \).

**Case 1:** \( \omega < \frac{(2n^3+1)\sigma}{2n^2(n+1)} \) and the solution lies at an extreme point. We will show that the solution in this case is \( q^n_T = 0 \) by direct comparison:

\[
P_i|_{q_i=1} - P_i|_{q_i=0} = \frac{1}{2m(n+1)^2} \left( 2n\omega \left( b(n+1) - (n^2 + 1)\sigma \right) - 2m(n+1)^2\alpha + \left( (2n^3 + 1)\sigma - 2n^2b(2n^2 + 1) \right)\sigma + 2\sigma \left( 2n(n+1)\omega - (2n^2 + 1)\sigma \right) \sum_{j \neq i} q_j \right) \quad (17)
\]

Our strategy will be to maximize expression (17) and show that it is negative. Expression (17) is decreasing in \( \alpha \) and \( \sum_{j \neq i} q_j \) (because \( \omega \leq \frac{(2n^3+1)\sigma}{2n^2(n+1)} \)), and increasing in \( \omega \). Thus, it is maximal when \( \sum_{j \neq i} q_j = 0, \alpha = 0, \) and \( \omega = \frac{(2n^3+1)\sigma}{2n^2(n+1)} \). Expression (17) evaluated at \( \sum_{j \neq i} q_j = 0, \omega = \frac{(2n^3+1)\sigma}{2n^2(n+1)}, \) and \( \alpha = 0 \) is

\[
-\frac{\sigma}{n} \left( (2n^3 + 2n^2 - 1)b - (2n^3 + 1)\sigma \right) \leq 0 \text{ with the inequality following from the assumption that } \sigma \leq b. \text{ So, if } \omega < \frac{(2n^3+1)\sigma}{2n^2(n+1)}, \text{ then } q^n_T = 0.
\]

**Case 2:** \( \omega \geq \frac{(2n^3+1)\sigma}{2n^2(n+1)} \). In this case the FOC is necessary and sufficient for non-signatory. Equilibrium is achieved the FOC for each of the \( n-k \) is satisfied simultaneously. Remember that each of the \( k \) signatories will choose the same abatement target \( q^s_s \). Further, because the non-signatories are identical they also choose the same abatement target \( q^n_n \). Our system of \( n-k \) equations can thus be reduced to a single FOC with in which these substitutions have been made:

\[
\frac{dP_i}{dq_i} = \frac{1}{m(n+1)^2} \left( n(n+1)\omega(b + \sigma) - (n+1)^2 m\alpha - n(2n+1)b\sigma - k \left( (2n^2 - 1)\sigma + 2n(n+1)\omega \right) q^s_s + \left( (2(k+1)n^2 + n-k)\sigma - 2(k+1)n(n+1)\omega \right) q^n_n \right) \quad (18)
\]

Because \( \omega \geq \frac{(2n^3+1)\sigma}{2n^2(n+1)} \), expression (17) is increasing in \( q^s_s \) and therefore maximal at \( q^s_s = 1 \). Expression (17) evaluated at \( q^s_s = 1 \) is increasing in \( k \)
and thus maximal at $k = n - 1$. Expression (17) evaluated at $q_s = 1$ and $k = n - 1$ is decreasing in $q_n$ and therefore maximal at $q_n = 0$. Expression (17) evaluated at $q_s = 1$, $k = n - 1$ and $q_n = 0$ is decreasing in $\alpha$ and thus maximal at $\alpha = 0$. Expression (17) evaluated at $q_s = 1$, $k = n - 1$, $q_n = 0$ and $\alpha = 0$ is:

\[
\frac{1}{m(n+1)^2} \left( (n(n+1)(b-(2n-3)\sigma)\omega - (n(2n+1)b+(n-1)(2n^2-1)\sigma)q_s \right) (19)
\]

Finally, solving expression (19) for $\omega$ we see that $(19) \leq 0$ iff

\[
\omega \leq \frac{(n(2n+1)b+(n-1)(2n^2-1)\sigma)\omega}{(n(n+1)(b-(2n-3)\sigma)} \triangleq \bar{\omega}^T.
\]

If this condition is met, then the marginal payoff of a nonsignatory is decreasing as its abatement target increases regardless of the value of $q_s$. So, $q_n^T = 0$. So, $q_n^T = 0$ if $\omega \leq \bar{\omega}^T \left( \frac{(2n^3+1)\sigma}{2n^2(n+1)} \right)$ regardless of the value of $q_s$.

### A.3.3 Signatories’ Subgame

The solution to the signatories’ subgame is characterized by the following conditions:

**FOC:**

\[
\frac{1}{m(n+1)^2} \left( kn(n+1)\omega(b+\sigma) - (n+1)^2 m\alpha - nb(2n+k-2)\sigma + \right.
\]

\[
\left. (k^2 - 2(k-1)n(2n-k+1) + 2n^3)\sigma - 2kn(n+1)(n-k+1)\omega \right) q_s \sigma q_s = 0
\]

**SOC:**

\[
\frac{\sigma}{m(n+1)^2} \left( (k^2 - 2(k-1)n(2n-k+1) + 2n^3)\sigma - 2kn(n+1)(n-k+1)\omega \right) \leq 0
\]

Note that the second order condition is satisfied iff:

\[
\omega \geq \frac{(k^2 - 2(k-1)n(2n-k+1) + 2n^3)\sigma}{2kn(n+1)(n-k+1)}
\]
**Case 1:** \( \omega < \frac{(k^2-2(k-1)n(2n-k+1)+2n^3)}{2kn(n+1)(n-k+1)} \sigma \). In this case the second order condition is not satisfied for any \( q_s \), so the solution is at an extreme point. We will again evaluate the alternatives via direct comparison. The expression for \( P_i|_{q_s=1} - P_i|_{q_s=0} \) is increasing in \( \omega \), decreasing in \( \alpha \), and increasing in \( k \). Evaluating this expression at \( \omega = \frac{(k^2-2(k-1)n(2n-k+1)+2n^3)}{2kn(n+1)(n-k+1)} \sigma \), \( \alpha = 0 \) and \( k = n \) yields \(-n(n+2)(b-\sigma)\sigma \leq 0\); so, \( q_s^T(k) = 0 \).

**Case 2:** \( \omega \geq \frac{(-4kn(1+n)+2n(1+n)^2+k^2(1+2n))}{2kn(1+n)(1-k+n)} \sigma \). In this case, the FOC is necessary and sufficient. Solving it for \( q_s \), we obtain:

\[
q_s^T(k) = \max \left\{ 0, \min \left\{ 1, \frac{f_1}{f_2} \right\} \right\}
\]

where,

\[
f_1 = -kn(n+1)(b+\sigma)\omega + (1+n)^2m\alpha + nb\sigma(2+2n-k)
\]

and

\[
f_2 = \left( k^2 - 2(k-1)n(2n-k+1) + 2n^3 \right) \sigma^2 - 2kn(n+1)(n-k+1)\omega \sigma
\]

### A.4 Smoothing Effect

#### A.4.1 Firm Behavior

Recall that to isolate the smoothing effect, we assume: \( x_i^M(\vec{q}) = x_i^A(\vec{q}) \) and \( \dot{x}_i^M(\vec{q}) = \frac{\sum_j x_j^i(\vec{q})}{n} \).

#### A.4.2 Nonsignatories’ Subgame

Nonsignatories simultaneously and unilaterally choose an abatement target to maximize their respective payoffs, anticipating the subsequent equilibrium in the goods and services market. The conditions that define the subgame equilibrium are:

\[
\text{FOC: } \frac{1}{4mn^2} \left( 2n^2(b+\sigma)\omega - 4n^2m\alpha - 3bn^2\sigma + \left( (1 - 2n + 4n^2)\sigma - 4n^2\omega \right) \sigma q_i - (n-1)\sigma^2 \sum_{j \neq i} q_j \right) = 0
\]

\[
\text{SOC } \frac{\sigma}{4n^2} \left( (1 - 2n + 4n^2)\sigma - 4n^2\omega \right) \leq 0
\]
The second order condition is satisfied iff \( \omega \geq \frac{(1-2n+4n^2)\sigma}{4n^2} \)

**Case 1:** \( \omega < \frac{(1-2n+4n^2)\sigma}{4n^2} \) and the solution must be at an extreme point. We will determine which by direct comparison:

\[
P_t|_{q_i=1} - P_t|_{q_i=0} = \frac{1}{8 m n^2} \left( 4 n^2 b \omega - 8 n^2 m \alpha - 6 n^2 b \sigma + \left( 1 - 2 n + 4 n^2 \right) \sigma^2 - 2 \left( n - 1 \right) \sigma^2 \sum_{j \neq i} q_j \right)
\]

Expression (20) is decreasing in \( \alpha \) and \( \sum_{j \neq i} q_j \) and increasing in \( \omega \); it is therefore maximal when \( \alpha = 0, \sum_{j \neq i} q_j = 0, \) and \( \omega = \frac{(1-2n+4n^2)\sigma}{4n^2} \). Expression (20) evaluated at these values is:

\[
\frac{\sigma}{8 m n^2} \left( (4 n^2 - 2 n - 1) - (2 n^2 - 1 + 2 n) \sigma \right) \leq 0
\]

with the inequality following because \( \sigma \leq \frac{1}{n} \). So, \( q_n^M = 0 \).

**Case 2:** \( \omega \geq \frac{(1-2n+4n^2)\sigma}{4n^2} \). In this case, the second order condition is satisfied; so the FOC is necessary and sufficient. Equilibrium is achieved the FOC for each of the \( n - k \) is satisfied simultaneously. By symmetry, we can substitute \( q_s \) for each signatory and \( q_n \) for each nonsignatory, reducing the system of equations to:

\[
\frac{dP_i}{dq_i} = \frac{1}{4 m n^2} \left( 2 n^2 (b + \sigma) \omega - 4 n^2 m \alpha - 3 n^2 b \sigma - k \left( n - 1 \right) \sigma^2 q_s + \left( (-k + k n + 3 n^2) \sigma - 4 n^2 \omega \right) \sigma q_n \right)
\]

Expression (21) is decreasing in \( q_s \) and \( \alpha \). Expression (21) evaluated at \( q_s = 0 \) and \( \alpha = 0 \) is increasing in \( k \). Since \( \omega \geq \frac{(1-2n+4n^2)\sigma}{4n^2} \), expression (21) evaluated at \( q_s = 0, \alpha = 0 \) and \( k = n - 1 \) is decreasing in \( q_n \). Expression (21) evaluated at \( q_s = 0, \alpha = 0, k = n - 1 \) and \( q_n = 0 \) is:

\[
\frac{1}{4 m n^2} \left( 2 n^2 (\sigma + b) \omega - 3 n^2 \sigma b \right), \text{ which is less than or equal to zero iff } \omega \leq \frac{3 \sigma}{2 (b + \sigma)}.
\]

So, if \( \omega \leq \frac{3 \sigma}{2 (1 + \sigma)} \), \( q_n^M = 0 \).
A.4.3 Signatories’ Subgame

The conditions that define the subgame equilibrium are:

\[ \text{FOC: } \frac{1}{4 mn^2} \left( 2 kn^2 \omega (b + \sigma) - 4 n^2 m \alpha - 3 b n^2 \sigma + \left( (k^2 - 2 kn + 4 n^2) \sigma^2 - 4 k n^2 \sigma \omega \right) q_s \right) = 0 \]

\[ \text{SOC: } \frac{\sigma}{4 n^2} \left( (k^2 - 2 kn + 4 n^2) \sigma - 4 kn^2 \omega \right) \leq 0 \]

Note that the second order condition is satisfied iff \( \omega \geq \frac{(k^2 - 2 kn + 4 n^2)}{4 k n^2} \).

**Case 1:** \( \omega < \frac{(k^2 - 2 kn + 4 n^2)}{4 k n^2} \) and the solution is at an extreme point. We will determine which by direct comparison:

\[ P_i|_{q_s=1} - P_i|_{q_s=0} = \]

\[ \frac{1}{8 m n^2} \left( 4 kn^2 b \omega - 8 n^2 m \alpha - 6 n^2 b \sigma + (k^2 - 2 kn + 4 n^2) \sigma^2 \right) \]

Expression (22) is decreasing in \( \alpha \) and increasing in \( \omega \). Expression (22) evaluated at \( \alpha = 0 \) and \( \omega = \frac{(k^2 - 2 kn + 4 n^2)}{4 k n^2} \) is decreasing in \( k \). Expression (22) evaluated at \( \alpha = 0 \), \( \omega = \frac{(k^2 - 2 kn + 4 n^2)}{4 k n^2} \) and \( k = 1 \) is:

\[ \frac{\sigma}{8 n^2} \left( (1 + 4 n^2 - 2 n) \sigma - (2 n^2 + 2 n - 1) b \right) \leq 0 \]

which is less than or equal to zero because \( \sigma \leq \frac{1}{\pi} \). So, \( q_s^M = 0 \).

Thus, if \( \omega < \frac{(k^2 - 2 kn + 4 n^2)}{4 k n^2} \), \( q_s^M (k) = 0 \).

**Case 2:** \( \omega \geq \frac{(k^2 - 2 kn + 4 n^2)}{4 k n^2} \). In this case, the SOC is satisfied; so, the FOC is necessary and sufficient. Solving if for \( q_s \) we obtain:

\[ q_s^M = \max \left\{ 0, \min \left\{ 1, \frac{n^2 (4 m \alpha + 3 b \sigma - 2 k \omega (\sigma + b))}{\sigma (4 n^2 - 2 n k - k^2) \sigma - 4 \omega k n^2} \right\} \right\} \]

A.4.4 Proof of Proposition 3

**Proof:** In general we know that the marginal benefit of abatement must be the same under the isolated smoothing effect and autarky because we
structured the analysis to ensure that production is the same and damages depend only on production. By contrast:

\[ MCA^M = MCA^A + (p(x^A_s) - p(\hat{x}^M)) \frac{\partial x^A}{\partial q_s} + p'(\hat{x}^M) \frac{\partial \hat{x}^M}{\partial q_s} (\hat{x}^M - x^A_s) \]

where, MCA is the decrease in domestic surplus that comes from increasing \( q_s \) and the arguments are suppressed for convenience. Recall that:

\[ \frac{d \hat{x}^M}{d q_s} = k \frac{d x^A}{d q_s}. \]

Also because \( q_s \geq q^M_n \Rightarrow x^M_i(q_s) \leq x^M_i(q^M_n) \). So, \( \hat{x}^M = k x^M_i(q_s) + (n-k) x^M_i(q^M_n) \) where, \( n \geq x^A_s \), and:

\[ (p(x^A_s) - p(\hat{x}^M)) + \frac{k}{n} p'(\hat{x}^M) (\hat{x}^M - x^A_s) \]

Notice that the last term is the difference between \( p(x^A_s) \) and a linear approximation thereto around the point \( \hat{x}^M \). Because \( p(i) \) is small for \( i \geq 2 \), this difference should be very small. So that:

\[ \frac{k}{n} [(p(x^A_s) - p(\hat{x}^M)) + p'(\hat{x}^M) (x^A_s - \hat{x}^M)] \approx 0 \]

Note that this inequality is exactly equal to zero when \( p \) is linear. And,

\[ (p(x^A_s) - p(\hat{x}^M)) + \frac{k}{n} p'(\hat{x}^M) (\hat{x}^M - x^A_s) \geq 0 \]

\[ \Rightarrow (p(x^A_s) - p(\hat{x}^M)) + \frac{k}{n} p'(\hat{x}^M) (\hat{x}^M - x^A_s) \frac{\partial x^A}{\partial q_s} \leq 0 \]

\[ \Rightarrow MCA^M(q_s) \leq MCA^A(q_s) \]

Finally, since \( MCA^A \geq MCA^M \) and \( MBA^A(q_s) = MBA^M(q_s) \) it follows that \( q^M(k) \geq q^A(k) \).

**A.5 Scale Effect**

**A.5.1 Firm Behavior**

To isolate the level effect, we assume: \( x^V_i(q) = \theta x^A_i(\bar{q}) \) and \( x^V_i(\bar{q}) = x^V_i(\bar{q}) \).
A.5.2 Nonsignatories’ Subgame

Nonsignatories simultaneously and unilaterally choose an abatement target to maximize their respective payoffs, anticipating the subsequent equilibrium in the goods and services market. The conditions that define the subgame equilibrium are:

**FOC:**

\[
\frac{1}{4m} \left( 2\theta \omega (b + \sigma) - 4m\alpha - (4 - \theta) \theta b\sigma + ((4 - \theta)\sigma - 4\omega) \theta \sigma q_i \right) = 0
\]

**SOC:**

\[
\frac{\theta \sigma}{4} ((4 - \theta)\sigma - 4\omega) \leq 0
\]

The second order condition is satisfied iff \( \omega \geq \frac{(4 - \theta)\sigma}{4} \).

**Case 1:** \( \omega < \frac{(4 - \theta)\sigma}{4} \) and the solution lies at one of the extreme points. We will determine which one by direct comparison:

\[
P_{i|q_i=1} - P_{i|q_i=0} = \frac{1}{8m} \left( 4\omega b\theta - 8m\alpha - \theta (4 - \theta)\sigma^2 - (4 - \theta)2b\sigma \right) \tag{23}
\]

Expression (23) is increasing in \( \omega \) and decreasing in \( \alpha \). Expression (23) evaluated at \( \omega = \frac{(4 - \theta)\sigma}{4} \) and \( \alpha = 0 \) is \( \frac{\theta(4 - \theta)\sigma(\sigma - b)}{8m} \leq 0 \) with the inequality holding when \( \theta \leq 4 \) and \( \sigma \leq b \). So, \( q_n^S = 0 \).

**Case 2:** \( \omega \geq \frac{(4 - \theta)\sigma}{4} \). In this case, the SOC is satisfied; so, the FOC is necessary and sufficient. Solving it \( q_i \) we obtain:

\[
q_n^S = \max \left\{ 0, \min \left\{ 1, \frac{2\theta \omega (\sigma + b) - 4m\alpha - (4 - \theta)\theta b\sigma}{\theta \sigma ((\theta - 4)\sigma + 4\omega)} \right\} \right\}
\]

So, if \( \omega \leq \frac{(4 - \theta)\theta b\sigma + 4m\alpha}{2\theta (\sigma + b)} \), then \( q_n^V = 0 \).

A.5.3 Signatories’ Subgame

Because \( x_i^S \) depends only on \( q_i \), the signatories’ problem is structurally the same as a nonsignatory’s if we were to substitute \( k\omega \) for \( \omega \). Thus, we know that the subgame equilibrium is:

\[
q_i^S(k) = \begin{cases} 
0 & \text{if } k \leq \frac{(4 - \theta)\theta b\sigma + 4m\alpha}{2\theta (\sigma + b)\omega} \equiv \bar{k}^S, \\
\frac{2\theta k\omega(\sigma + b) - 4m\alpha - (4 - \theta)\theta b\sigma}{\theta \sigma ((\theta - 4)\sigma + 4k\omega)} & \text{if } k \in [\bar{k}_i^S, \bar{k}^S], \\
1 & \text{if } k \geq \frac{(4 - \theta)\theta b\sigma (\sigma - b)}{2\omega (\sigma + b)} \equiv \bar{k}^V.
\end{cases}
\]
A.5.4 Proof of Proposition 4

Proof: Under isolated scale effects: \( MBA^S = \theta MBA^A \) because damages are linear in production and \( x_n^S = \theta x_n^A \) and \( x_s^S = \theta x_s^A \). By contrast, the marginal cost of abatement is:

\[
MCA^S = \theta MCA^A + (1 - \theta) \alpha + \theta (p(x_s^A) - p(\theta x_s^A)) \frac{dx_s^A}{dq_i}.
\]

where \( MCA \) is the decrease in domestic surplus of a signatory that comes from increasing \( q_s \). But \( (1 - \theta) \alpha \leq 0 \) because \( \theta \geq 1 \) and \( \alpha \geq 0 \). Further, \( (p(x_s^A) - p(\theta x_s^A)) \frac{dx_s^A}{dq_i} \leq 0 \) because \( \theta > 1 \), \( p' < 1 \) and \( \frac{dx_s^A}{dq_i} \leq 0 \). So:

\[
MCA^S(q_s) \leq \theta MCA^A(q_s).
\]

It follows then that, \( q_s^S \geq q_s^A \).

The proof is the second part is straight forward. If \( k_0^A \) is the value of \( k \) for which \( q_s^A(k) = 0 \) and \( k_1^S \) is the value of \( k \) for which \( q_s^S(k) = 1 \), then:

\[
k_0^A - k_1^S = \frac{\left((4 m \theta (b - \sigma) - 4 m (b + \sigma)) \alpha + (4 - \theta) \theta \sigma^3 - 3 b \theta \sigma^2 + b^2 \theta \sigma (\theta - 1)\right)}{2 \omega \theta (b - \sigma)}
\]

(24)

The denominator is positive for \( \sigma < \frac{b}{n-1} \); so we need only show that the numerator is nonnegative. The numerator is increasing in \( \alpha \) because \( \theta \geq \frac{b+\sigma}{b-\sigma} \geq \frac{b+\sigma}{b-\sigma} \). Evaluated at \( \alpha = 0 \) expression (24) is:

\[
k_0^A - k_1^S = \frac{\left((4 - \theta) \theta \sigma^3 - 3 b \theta \sigma^2 + b^2 \theta \sigma (\theta - 1)\right)}{2 \omega \theta (b - \sigma)} \geq 0
\]

(25)

So by the logic of Section 3, since \( k_0^A \geq k_1^S \), the IEA will have more participation under autarky than under trade with the scale effect isolated.

A.6 Leakage Effect

A.6.1 Firm Behavior

To isolate the level effect, we assume: \( x_t^L(q) = \frac{b - \sigma q}{2m} + \lambda (\bar{q} - q) \), where \( \bar{q} = \frac{\sum_i q_i}{n} \) and \( \dot{x}_t^L(\bar{q}) = x_t^L(\bar{q}) \).
A.6.2 Nonsignatories’ Subgame

Nonsignatories simultaneously and unilaterally choose an abatement target to maximize their respective payoffs, anticipating the subsequent equilibrium in the goods and services market. The subgame equilibrium is characterized by the following conditions:

\[
\text{FOC: } \frac{1}{4mn^2} \left( 2n^2 (\sigma + b) \omega - 4mn^2 \alpha - 3n^2 b \sigma - 2n(n-1)b \lambda + \right. \\
\left. \left( 4\lambda^2 m^2 (n-1) + 8 \lambda mn \omega - 2 \lambda mn \sigma \right) \sum_{j \neq i} q_j + \right. \\
\left. \left( 3n^2 \sigma^2 + 4n(n-1) \lambda m \sigma - (8n(n-1) \lambda m + 4n^2 \sigma) \omega - ight. \\
\left. \left. 4\lambda^2 m^2 (n^2 - 2n + 1) \right) q_i \right) = 0
\]

\[
\text{SOC: } \frac{1}{4mn^2} \left( 3n^2 \sigma^2 + 4n(n-1) \lambda m \sigma - (8n(n-1) \lambda m + 4n^2 \sigma) \omega - \\
\left. \left. 4\lambda^2 m^2 (n^2 - 2n + 1) \right) q_i \leq 0
\]

Notice that second order conditions are satisfied as long as \( \omega \geq \frac{3\sigma n - 2m(n-1)\lambda}{4n} \) and that this condition holds for all positive \( \omega \) if \( \lambda \geq \frac{3\sigma n}{2m(n-1)} \). For convenience we will assume that these conditions hold; so the FOC is necessary and sufficient.

Notice that the left-hand side of the FOC (referred to as FOC hereafter) is increasing in \( q_s \) and decreasing in \( \alpha \) and therefore maximal at \( q_s = 1 \) and \( \alpha = 0 \).

**Case 1:** If \( k \leq \frac{n \sigma (2(n-1) m \lambda - 2n \omega + 3n \sigma)}{2\lambda m (4n\omega + 2\lambda m(n-1) - n\sigma)} \), then FOC evaluated at \( q_s = 1 \) and \( \alpha = 0 \) is increasing in \( q_n \) and maximal at \( q_n = 1 \). FOC evaluated at these values is nonpositive if \( \omega \leq \frac{2(n-1) m \lambda + 3 \sigma n}{n} \) and \( q_n^L = 0 \).

**Case 2:** If \( k \leq \frac{n \sigma (2(n-1) m \lambda - 2n \omega + 3n \sigma)}{2\lambda m (4n\omega + 2\lambda m(n-1) - n\sigma)} \), then FOC evaluated at \( q_s = 1 \) and \( \alpha = 0 \) is decreasing in \( q_n \) and maximal at \( q_n = 0 \). FOC evaluated at these values is nonpositive if: \( \omega \leq \frac{3n^2 b \sigma + 2n(n-1) m \lambda (\sigma + b) - 4(n^2 - 2n + 1) \lambda^2 m^2}{2n(n(\sigma + b) + 4(n-1) \lambda m)} \) and \( q_n^L = 0 \).
A.6.3 Signatories’ Subgame

The conditions that define the subgame equilibrium are:

**FOC:**
\[
\frac{1}{4m n^2} \left( 2 k n^2 \omega (\sigma + b) - 3 n^2 \sigma^2 - 4 n^2 m \alpha - 2 n (n - k) b \lambda m + \right.
\]
\[
\left. 3 n^2 \sigma^2 + 4 n (n - k) \lambda m \sigma - (4 k n^2 \sigma + 8 n k (n - k) \lambda m) \omega - 4 (n - k)^2 \lambda^2 m^2 \right) q_s = 0
\]

**SOC:**
\[
\frac{1}{4m n^2} \left( 3 n^2 \sigma^2 + 4 n (n - k) \lambda m \sigma - (4 k n^2 \sigma + 8 n k (n - k) \lambda m) \omega - 4 (n - k)^2 \lambda^2 m^2 \right) \leq 0
\]

Note that the SOC are satisfied for the \( \omega \) values that guarantee \( q_n^L = 0 \). So, by assumption the FOC is necessary and sufficient. Solving it for \( q_s \) we obtain:

\[
q_s^L(k) = \max \left\{ 0, \min \left\{ 1, \frac{\ell_1}{\ell_2} \right\} \right\}
\]

where:

\[
\ell_1 = 3 n^2 \sigma^2 + 4 n (n - k) \lambda m \sigma - (4 k n^2 \sigma + 8 n k (n - k) \lambda m) \omega - 2 k n^2 \omega (\sigma + b)
\]

and

\[
\ell_2 = 3 n^2 \sigma^2 + 4 n (n - k) \lambda m \sigma - (4 k n^2 \sigma + 8 n k (n - k) \lambda m) \omega - 4 (n - k)^2 \lambda^2 m^2
\]

A.6.4 Proof of Proposition 5

**Proof:** Substituting the values from Sections A.2 and A.6, we have:

\[
MBA^L - MBA^A = -\frac{2 \omega \lambda k (n - k)}{n} q_s < 0
\]

\[
MCA^L - MCA^A = \frac{2 n (n - k) m \lambda}{4 m n^2} \left( (n - k) m \lambda - \sigma \right) + b) \geq \frac{2 n (n - k) m \lambda}{4 m n^2} \left( -2 \sigma + b \right) \geq \frac{2 n (n - k) m \lambda}{4 m n^2} \left( -\frac{2 b}{n - 1} + b \right) \geq 0
\]

where MCA and MBA are the decrease in domestic surplus and pollution damages, respectively, that come from increasing \( q_s \). So, \( MBA^L \leq MBA^A \) and \( MCA^L \geq MCA^A \Rightarrow q_s^L(k) \leq q_s^A(k) \).
The proof is the second part is straightforward. If $k_1^A$ is the value of $k$ for which $q_s^A(k) = 1$ and $k_0^L$ is the value of $k$ for which $q_s^L(k) = 1$, then:

$$k_0^L - k_1^A = \frac{\lambda}{2\omega(b - \sigma)(b \lambda m + \omega n(b + \sigma))} \left( 2nm(b - \sigma)\lambda - 3n\sigma^2(b - \sigma) - 8nm\sigma\alpha \right) \omega - \left( 3\sigma bm(b - \sigma) + 4\alpha m^2b \right)$$

(26)

The denominator of this expression is positive and the increasing in $\omega$ because $\lambda \geq \frac{3n\sigma}{2m(n-1)}$ and $\alpha \leq \frac{3(b-\sigma)^2}{m}$. Setting the numerator equal to zero and solving for $\omega$, we get:

$$\omega^* = \frac{(3\sigma bm(b - \sigma) + 4\alpha m^2b)}{(2nm(b - \sigma)\lambda - 3n\sigma^2(b - \sigma) - 8nm\sigma\alpha)}$$

So, $k_0^L \geq k_1^A$ for $\omega \geq \omega^*$, which implies that there will be more participation in an IEA under trade with isolated leakage effects than under autarky for these parameter values.
References


