Playsheet 23
Fibonacci Numbers

Directions: Work together on each problem; do not delegate different problems to different people. Submit one neatly written write-up per group. Remember to use complete sentences as appropriate and to show your work.

The Fibonacci numbers are 1, 1, 2, 3, 5, 8, 13, ... Each number is found by adding the previous two numbers in the sequence. We say that the Fibonacci numbers are defined recursively. That is, each number is defined in terms of some combination of the previous numbers in the sequence. If we let $F_n$ represent the $n$th number in the Fibonacci Sequence, the recursive rule for the Fibonacci Sequence is

$$F_n = F_{n-1} + F_{n-2}.$$ 

To completely define the sequence we must also say that $F_1 = F_2 = 1$. These are called the seeds.

1. Find each of the following.
   
   (a) $F_8 =$
   
   (b) $F_9 =$
   
   (c) $F_{10} =$
   
   (d) $F_{11} =$

2. If you know that $F_{22} = 17711$ and $F_{23} = 28657$, then
   
   (a) $F_{24} =$
   
   (b) $F_{25} =$

3. Given the seeds $G_1 = 1, G_2 = 2$ and the recursive formula $G_{n+1} = G_{n-1} - 2G_n$, find each of the following.
   
   (a) $G_3 =$
   
   (b) $G_4 =$
   
   (c) $G_5 =$

4. If you know that $G_{11} = -3771$ and $G_{12} = 9104$, what is $G_{13}$?

5. Suppose there is a sequence defined recursively by $H_n = H_{n-2} + 2H_{n-1}$. You don't know $H_1$ or $H_2$, but you are told that $H_{11} = w$ and $H_{12} = t$.
   
   (a) What is $H_{14}$?
   
   (b) What is $H_{10}$?