Solutions to Homework Assignment 5

1. \[5^t = 7\]
   \[\ln(5^t) = \ln(7)\]
   \[t \ln(5) = \ln(7)\]
   \[t = \frac{\ln(7)}{\ln(5)}\]

2. \[130 = 10^t\]
   \[\ln(130) = \ln(10^t)\]
   \[\ln(130) = t \ln(10)\]
   \[t = \frac{\ln(130)}{\ln(10)}\]

3. \[2 = (1.02)^t\]
   \[\ln(2) = \ln[(1.02)^t]\]
   \[\ln(2) = t \ln(1.02)\]
   \[t = \frac{\ln(2)}{\ln(1.02)}\]

4. \[10 = 2^t\]
   \[\ln(10) = \ln(2^t)\]
   \[\ln(10) = t \ln(2)\]
   \[t = \frac{\ln(10)}{\ln(2)}\]

5. \[100 = 25(1.5)^t\]
   \[4 = (1.5)^t\]
   \[\ln(4) = \ln[(1.5)^t]\]
   \[\ln(4) = t \ln(1.5)\]
   \[t = \frac{\ln(4)}{\ln(1.5)}\]

6. \[50 = 10 \cdot 3^t\]
   \[5 = 3^t\]
   \[\ln(5) = \ln(3^t)\]
   \[\ln(5) = t \ln(3)\]
   \[t = \frac{\ln(5)}{\ln(3)}\]

7. \[a = b^t\]
   \[\ln(a) = \ln(b^t)\]
   \[\ln(a) = t \ln(b)\]
   \[t = \frac{\ln(a)}{\ln(b)}\]

I will use this fact as needed from now on in this solution set.

8. \[t = \frac{\ln(10)}{\ln(e)}\]
   \[t = \ln(10)\]

(See number 7 with \(a = 10\) and \(b = e\).)

9. \[5 = 2e^t\]
   \[2.5 = e^t\]
   \[t = \frac{\ln(2.5)}{\ln(e)}\]
   \[t = \ln(2.5)\]

10. \[e^{3t} = 100\]
    \[\ln(e^{3t}) = \ln(100)\]
    \[3t = \ln(100)\]
    \[t = \frac{\ln(100)}{3}\]

11. \[10 = 6e^{0.5t}\]
    \[\frac{5}{3} = e^{0.5t}\]
    \[\ln\left(\frac{5}{3}\right) = \ln(e^{0.5t})\]
    \[\ln\left(\frac{5}{3}\right) = 0.5t\]
    \[t = 2 \ln\left(\frac{5}{3}\right)\]
12. 
\[ 40 = 100e^{-0.03t} \]
\[ 0.4 = e^{-0.03t} \]
\[ \ln(0.4) = \ln(e^{-0.03t}) \]
\[ \ln(0.4) = -0.03t \]
\[ t = \frac{\ln(0.4)}{-0.03} \].

13. 
\[ B = Pe^{rt} \]
\[ \frac{B}{P} = e^{rt} \]
\[ \ln \left( \frac{B}{P} \right) = \ln(e^{rt}) \]
\[ \ln \left( \frac{B}{P} \right) = rt \]
\[ t = \frac{1}{r} \cdot \ln \left( \frac{B}{P} \right). \]

14. 
\[ 2P = Pe^{0.3t} \]
\[ 2 = e^{0.3t} \]
\[ \ln(2) = \ln(e^{0.3t}) \]
\[ \ln(2) = 0.3t \]
\[ t = \frac{\ln(2)}{0.3}. \]

15. 
\[ 7 \cdot 3^t = 5 \cdot 2^t \]
\[ \ln(7) + t \ln(3) = \ln(5) + t \ln(2) \]
\[ t \ln(3) - t \ln(2) = \ln(5) - \ln(7) \]
\[ t[\ln(3) - \ln(2)] = \ln(5) - \ln(7) \]
\[ t = \frac{\ln(5) - \ln(7)}{\ln(3) - \ln(2)}. \]

16. 
\[ 5e^{3t} = 8e^{2t} \]
\[ \frac{e^{3t}}{e^{2t}} = \frac{8}{5} \]
\[ e^t = 1.6 \]
\[ \ln(e^t) = \ln(1.6) \]
\[ t = \ln(1.6). \]

17. The initial quantity is 5 units and the growth rate is 7%.

18. The initial quantity is 7 units and the decay rate is 8%.

19. The initial quantity is 3.2 units and the growth rate is 3%; it is continuous.

20. The initial quantity is 15 units and the decay rate is 7%; it is continuous.

21. \[ P = e^{0.08t} = (e^{0.08})^t = 1.083^t. \] \[ Q = e^{-0.3t} = (e^{-0.3})^t = 0.741^t. \]

23. (a) Town D is growing fastest with a continuous growth rate of 12%
     (b) Town C is the largest now; its population is 1200.
     (c) Only town B is decreasing in size; its continuous growth rate is negative.

26. 
\[ P = 15e^{0.25t} \]
\[ = 15(e^{0.25})^t \]
\[ = 15(1.284)^t. \]
This is growth.

27. 
\[ P = 2e^{-0.5t} \]
\[ = 2(e^{-0.5})^t \]
\[ = 2(0.607)^t. \]
This is decay.
28. 

\[ P = P_0e^{0.2t} \]
\[ = P_0(e^{0.2})^t \]
\[ = P_0(1.221)^t. \]

This is growth.

29. 

\[ P = 7e^{-\pi t} \]
\[ = 7(e^{-\pi})^t \]
\[ = 7(0.0432)^t. \]

This is decay.

30. We need to write \(1.5 = e^k\) for some \(k\). Such a \(k\) satisfies \(\ln(1.5) = k\), so \(k \approx 0.405\). Thus

\[ 15(1.5)^t = 15(e^{0.405})^t \]
\[ = 15e^{0.405t}. \]

31. We need a \(k\) such that \(e^k = 1.7\), so \(k = \ln(1.7) \approx 0.531\). Thus

\[ P = 10(1.7)^t \]
\[ = 10(e^{0.531})^t \]
\[ = 10e^{0.531t}. \]

32. We need \(0.9 = e^k\), so \(k = \ln(0.9) \approx -0.105\). Thus

\[ P = 174(0.9)^t \]
\[ = 174(e^{-0.105})^t \]
\[ = 174e^{-0.105t}. \]

33. We need \(e^k = 0.55\), so \(k = \ln(0.55) \approx -0.598\). Thus

\[ P = 4(0.55)^t \]
\[ = 4(e^{-0.598})^t \]
\[ = 4e^{-0.598t}. \]

35. (a) The annual percent decay rate is 12%; notice that \(1 - 0.88 = 0.12\).

(b) We need \(e^k = 0.88\), so \(k = \ln(0.88) \approx -0.128\). Thus

\[ P = 25(0.88)^t \]
\[ = 25(e^{-0.128})^t \]
\[ = 25e^{-0.128t}. \]

This gives a continuous percent decay rate of 12.8%.

40. We have

\[ P = e^{0.08t} \]
\[ = (e^{0.08})^t \]
\[ = 1.0833^t. \]

Thus, a continuous rate of 8% is equivalent to an annual rate of 8.33%.

41. We have \(P = 1.1^t\), so to convert we need \(e^k = 1.1\), and \(k = \ln(1.1) \approx 0.0953\). This number, 9.53%, is the equivalent continuous growth rate.