Solutions to Homework Assignment 10

MATH 139-01 and -02
1-4, 6, 8-10, 12-15, 18, 21, 22, 25

1. (a) The average velocity is \( \frac{5^2 - 3^2}{5 - 3} = 8 \) feet per second.

(b) From 3 to 3.1, the average velocity is \( \frac{3.1^2 - 3^2}{3.1 - 3} = 6.1 \) feet per second. From 3 to 3.01, the average velocity is \( \frac{3.01^2 - 3^2}{3.01 - 3} = 6.01 \). From 3 to 3.0001, it is \( \frac{3.0001^2 - 3^2}{3.0001 - 3} = 6.00001 \). It appears that the instantaneous velocity is 6 ft/s.

2. (a) The total change in size is \( 2^6 - 2^0 = 63 \) mm³.

(b) The average rate of change is \( \frac{63}{6} = 10.5 \) mm³/month.

(c) From 6 to 6.1 months, the average rate of change is \( \frac{2^{6.1} - 2^6}{6.1 - 6} \approx 45.935 \). From 6 to 6.001, it is \( \frac{2^{6.001} - 2^6}{6.001 - 6} \approx 44.3768 \). From 6 to 6.00001, it is \( \frac{2^{6.00001} - 2^6}{6.00001 - 6} \approx 44.36157 \). From 5.99999 to 6, it is \( \frac{2^{5.99999} - 2^6}{6 - 5.99999} \approx 44.36127 \). It looks like the rate of growth is about 44.36 mm³ per month at 6 months.

3. The only point at which the slope is 0 is E. Points C and F are the only two at which the slope is negative, and the slope at F is steeper than that at C. Thus, F must correspond to −3 and C to −1. It also appears that the slope at A is least (and therefore 1/2) and the slope at D is greatest (and therefore 2). This means that the slope at B must be 1.

4. The average velocity over \([0, 0.8]\) is \( \frac{6.5 - 0}{0.8 - 0} = 8.125 \) feet per second. To estimate the velocity at \( t = 0.2 \), we will take the smallest interval near \( t = 0.2 \) that we can. I will first approximate from above: \( \frac{1.8 - 0.5}{0.4 - 0.2} = 6.5 \). The approximation from below is \( \frac{0.5 - 0}{0.2 - 0} = 2.5 \). These are quite a bit different! Their average is 4.5, so that is a reasonable approximation of the rate of change at \( t = 0.2 \). Another way to approximate would be to “surround” \( t = 0.2 \); that is, to compute the average rate of change from 0 to 0.4. This is \( \frac{1.8 - 0}{0.4 - 0} = 4.5 \).

6. The slope of the graph is positive at A and D. It is negative at C and F, and 0 at B and E. It appears to have the greatest slope at A and the least at F.

8. I will use 2 to 2.01 first: \( \frac{2^{2.01} - 2^2}{2.01 - 2} \approx 40.561 \). Using 2 to 2.0001 gives \( \frac{2^{2.0001} - 2^2}{2.0001 - 2} \approx 40.239 \). (The exact value is closer to 40.236 – not bad!)

9. I will use a very small interval to begin: \( f'(2) \approx \frac{3^{2.0001} - 3^2}{2.0001 - 2} \approx 9.888 \). This is the slope of a secant line through two points very close together, one of which is at \( t = 2 \). It should be close to the slope of the tangent line at \( t = 2 \), which is \( f'(2) \).

10. I will compute the slope of a secant line that is pretty close to a tangent line at \( t = 0 \) by choosing a small interval near 0; say, \( [0, 0.0001] \). The average rate of change over this interval is \( \frac{200(1.05)^{0.0001} - 200(1.05)^0}{0.0001 - 0} \approx 97.58 \).

12. (a) i. \( \frac{3(1 + 0.1)^2 - 3(1)^2}{1.1 - 1} = 6.3 \).

ii. \( \frac{3(1.01)^2 - 3(1)^2}{1.01 - 1} = 6.03 \).
iii. \[
\frac{3(1.001)^2 - 3(1)^2}{1.001 - 1} = 6.003.
\]
(b) It appears that the instantaneous velocity at time \( t = 1 \) s is 6 m/s.

13. The tangent line at \( d \) is horizontal, so \( f'(d) = 0 \). The tangent lines at \( a \) and \( e \) both slope downward, but the one at \( e \) does so more steeply; thus, \( f'(a) = -0.5 \) and \( f'(e) = -2 \). Likewise, the tangent lines at \( b \) and \( c \) both rise, but the one at \( c \) does so more steeply; therefore, \( f'(b) = 0.5 \) and \( f'(c) = 2 \).

14. (a) From the graph (below), it appears that \( f'(1) \) is negative.

(b) I will use a width of 0.001:
\[
\frac{(2 - (1.001)^3) - (2 - 1^3)}{1.001 - 1} = -3.003001.
\]
\( f'(1) \) is probably about \(-3\), which is negative (as expected).

15. (a) It appears that \( g'(2) \) should be negative. In fact, since this function decays exponentially, we would expect (without even looking at the graph!) that \( f'(x) \) is negative for all \( x \).

(b) I will use a width of 0.001:
\[
\frac{0.8^{2.001} - 0.8^2}{2.001 - 2} \approx -0.143,
\]
which is negative.

18.

21. We need an equation for the tangent line. We are told its slope (\( f'(4) = 1.5 \)) and one point on it (\( f(4) = 25 \)), so we can find an equation: \( y - 25 = 1.5(x - 4) \). Thus, \( y = 1.5x + 19 \). For \( C, x = 3.9 \), so \( C \) has coordinates \((3.9, 1.5(3.9) + 19) = (3.9, 24.85)\). Point \( B \) has coordinates \((4.2, 1.5(4.2) + 19) = (4.2, 25.3)\). Point \( A \) was the given point with coordinates \((4, 25)\).
22. (a) $g(2) = 5$.

(b) $g'(2) = \frac{5 - 5.02}{2 - 1.95} = -0.4$. The line shown is the tangent line to $g$ at $(2,5)$, so its slope (which we just computed) is $g'(2)$.

25. I will use an interval width of 0.0001. $f'(1) \approx \frac{1.0001 \ln(1.0001) - 1 \ln(1)}{1.0001 - 1} \approx 1$. $f'(2) \approx \frac{2.0001 \ln(2.0001) - 2 \ln(2)}{2.0001 - 2} \approx 1.693$. Since the slope of the tangent line is getting steeper, the function must be concave up at this point.