Solutions to Homework Assignment 14

4. The sequence of graphs below is zooming in on the graph of \( \frac{5^x - 1}{x} \) at \( x = 0 \). It looks like the limit is about 1.60944.

5. With \( h = 0.01 \), I have \( \frac{3.01^3 - 27}{0.01} \approx 27.09 \). With \( h = 0.0001 \), this becomes \( \frac{3.0001^3 - 27}{0.0001} \approx 27.0009 \).

Thus, \( \lim_{h \to 0} \frac{(3 + h)^3 - 27}{h} = 27 \), to within one decimal place.

6. With \( h = 0.01 \), we get \( \frac{e^{1.01} - 1}{0.01} \approx 1.965 \). With \( h = 0.0001 \), we get \( \frac{e^{0.0001} - 1}{0.0001} \approx 1.9459 \approx 1.9 \).

7. With \( h = 0.01 \), we get \( \frac{e^{0.01} - e}{0.01} \approx 2.7319 \). With \( h = 0.0001 \), we get \( \frac{e^{0.0001} - e}{0.0001} \approx 2.7182 \approx 2.7 \).

9. \( f \) appears to be continuous on both intervals; there are no breaks or jumps in the graph.

10. \( f \) does not appear to be continuous on \( 0 \leq x \leq 2 \) since there is a jump in the graph, but it does appear to be continuous on \( 0 \leq x \leq 0.5 \).

11. \( f \) is not continuous on \([0, 2]\) since there is a break in its graph; it is continuous on \([0, 0.5]\).

12. \( f \) is not continuous on \([0, 2]\) since the function value and the limit value do not agree at \( x = 1 \). However, \( f \) is continuous on \([0, 0.5]\).

13. Since \( x + 2 \) is a polynomial, it is continuous on the interval.

14. Since \( 2^x \) is an exponential function, it is continuous on the interval.

15. Since \( x^2 + 2 \) is a polynomial, it is continuous on the interval.

16. The only problem with \( f \) is at \( x = 1 \); since \( x = 1 \) is not in the interval, \( f \) is continuous on the interval.

17. This function is not defined at \( x = 1 \); since \( x = 1 \) is in the interval, \( f \) is not continuous on this interval.

18. \( f \) is continuous on the interval; it has no “tricky points”.

19. This is not continuous; every time someone is born, dies, enters, or leaves, the function value changes by 1. This means that the graph jumps by 1 whenever it changes, so there are lots of gaps in the graph.

20. The weight gain should be continuous. The baby won’t suddenly gain or lose a pound; it takes a little time.

21. This is not continuous; it is like number 19. A pair of pants is not “made” until it is complete; after that, the function value jumps by 1.
22. Even in stop-and-go traffic, the stopping and going is not instantaneous; there is some time for acceleration. This is still continuous.

23. This will not be continuous since it will jump when you cross a time zone.

24. 

\[
\lim_{h \to 0} \frac{5(x + h) - 5x}{h} = \lim_{h \to 0} \frac{5x + 5h - 5x}{h} = \lim_{h \to 0} \frac{5h}{h} = \lim_{h \to 0} (5) = 5,
\]

as expected.

25. 

\[
\lim_{h \to 0} \frac{[3(x + h) - 2] - [3x - 2]}{h} = \lim_{h \to 0} \frac{3x + 3h - 2 - 3x + 2}{h} = \lim_{h \to 0} \frac{3h}{h} = \lim_{h \to 0} 3 = 3.
\]

26. 

\[
\lim_{h \to 0} \frac{[(x + h)^2 + 4] - [x^2 + 4]}{h} = \lim_{h \to 0} \frac{x^2 + 2hx + h^2 + 4 - x^2 - 4}{h} = \lim_{h \to 0} \frac{2hx + h^2}{h} = \lim_{h \to 0} \frac{h(2x + h)}{h} = \lim_{h \to 0} (2x + h) = 2x.
\]

27. 

\[
\lim_{h \to 0} \frac{3(x + h)^2 - 3x^2}{h} = \lim_{h \to 0} \frac{3(x^2 + 2hx + h^2) - 3x^2}{h} = \lim_{h \to 0} \frac{6hx + 3h^2}{h} = \lim_{h \to 0} \frac{h(6x + 3h)}{h} = \lim_{h \to 0} (6x + 3h) = 6x.
\]
28. 
\[
\lim_{h \to 0} \frac{[5(x + h)^2 + 1] - [5x^2 + 1]}{h} = \lim_{h \to 0} \frac{[5(x^2 + 2hx + h^2) + 1] - 5x^2 - 1}{h} \\
= \lim_{h \to 0} \frac{10hx + 5h^2}{h} \\
= \lim_{h \to 0} \frac{h(10x + 5h)}{h} \\
= \lim_{h \to 0} (10x + 5h) \\
= 10x.
\]

29. 
\[
\lim_{h \to 0} \frac{[2(x + h)^2 + (x + h)] - [2x^2 + x]}{h} = \lim_{h \to 0} \frac{2(x^2 + 2hx + h^2) + x + h - 2x^2 - x}{h} \\
= \lim_{h \to 0} \frac{4hx + 2h^2 + h}{h} \\
= \lim_{h \to 0} \frac{h(4x + 2h + 1)}{h} \\
= \lim_{h \to 0} (4x + 2h + 1) \\
= 4x + 1.
\]