1. There are two inflection points.

2. There are three inflection points.

3. There are two inflection points.

4. There is one inflection point.

5. \( f'(x) = 4x^3 + 3x^2 - 6x \) and \( f''(x) = 12x^2 + 6x - 6 = 6(2x^2 + x - 1) \). The candidates for inflection points are the points at which \( f''(x) = 0 \), so we solve \( 2x^2 + x - 1 = 0 \). We get \( (2x - 1)(x + 1) = 0 \), so \( x = 1/2 \) or \( x = -1 \). Since \( f''(-2) = 12(-2)^2 + 6(-2) - 6 = 30 > 0 \), the graph of \( f \) is concave up to the left of \( x = -1 \). Since \( f''(0) = -6 < 0 \), the graph is concave down between \( x = -1 \) and \( x = 1/2 \). Thus, \( x = -1 \) is an inflection point. Since \( f''(1) = 12 > 0 \), the graph is again concave up to the left of \( x = 1/2 \), so \( x = 1/2 \) is also an inflection point.

6. \( f'(x) = 2x - 5 \), so the only critical number is \( x = 5/2 \). From the graph, we can see that this is a local minimum. Since \( f''(x) = 2 \), there are no inflection points.

7. \( f'(x) = 6x^2 + 6x - 36 = 6(x^2 + x - 6) = 6(x + 3)(x - 2) \). Therefore, the critical numbers are \( x = -3 \) and \( x = 2 \). We can see from the graph that \( x = -3 \) is a local maximum and \( x = 2 \) is a local minimum. \( f''(x) = 12x + 6 \), so the only second-order critical number is \( x = -1/2 \), we can see from the graph that \( x = -1/2 \) is in fact an inflection point since the graph changes concavity there.

8. \( f'(x) = 12x^3 - 12x^2 = 12x^2(x - 1) \), so \( x = 0 \) and \( x = 1 \) are both critical numbers. The graph indicates that \( x = 0 \) gives neither a local maximum nor a local minimum, but \( x = 1 \) gives a local minimum.

9. \( f'(x) = 4x^3 - 16x = 4x(x^2 - 4) - 4x(x - 2)(x + 2) \). The critical numbers are \( x = 0 \), \( 2 \), and \( -2 \). The graph indicates that \( x = -2 \) and \( x = 2 \) both give local minima, while \( x = 0 \) gives a local maximum. \( f''(x) = 12x^2 - 16 = 4(3x^2 - 1) \). This is zero when \( 3x^2 = 1 \), or \( x^2 = 1/3 \). The inflection points are at \( x = \pm \sqrt{1/3} \), as we can see on the graph.

10. \( f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3) \), so the critical numbers are \( x = 0 \) and \( x = 3 \). The graph indicates that \( x = 0 \) gives neither a maximum nor a minimum, but \( x = 3 \) gives a local minimum. \( f''(x) = 12x^2 - 24x = 12x(x - 2) \), so the candidates for inflection points are \( x = 0 \) and \( x = 2 \). We can see from the graph that both of these are in fact inflection points.

11. \( f'(x) = 15x^4 - 15x^2 = 15x^2(x^2 - 1) = 15x(x - 1)(x + 1) \). The critical numbers are \( x = 0 \) and \( x = \pm 1 \). We can see from the graph that \( x = -1 \) gives a local maximum, \( x = 0 \) gives neither a maximum nor a minimum, and \( x = 1 \) gives a local minimum. \( f''(x) = 60x^3 - 30x = 30x(2x^2 - 1) \). The candidates for inflection points are \( x = 0 \) and \( x = \pm \sqrt{1/2} \). We see from the graph that in fact all are inflection points.
14. (a) The population does level off at around 2000 wabbits.

(b) It looks like the population grew most rapidly around 1786 (around 12 years after the rabbits were left), when the population was around 1000.

(c) The inflection point is around (12, 1000); that is where the population is growing most rapidly.

(d) The rabbits breed exponentially while they can, but the island has limited resources and cannot support too large a population.

15. The graphs are below.
26. We need a function that is increasing but concave down until $x_1$, at which point it has a horizontal tangent line, increasing and concave up until $x_2$, increasing and concave down until $x_3$, and then decreasing and concave down. See my graph below; it has $x_1 = 0$, $x_2 \approx 2.7$, and $x_3 = 4$.

27. We need a function that is decreasing everywhere but changes from concave up to concave down to concave up and then back to concave down.

28. We need a graph that is increasing everywhere except for a flat spot at $x_1$. We also need to change from concave down to concave up and then back to concave down.

29. We need a straight line with slope 2 until $x_1$, then we need the graph to be increasing and concave up.