1. After the first deposit, the balance is $1000. After the second deposit, the balance is $1000 + 1000(0.085) = 1000 + 1000(1.085)$ dollars. (That is the second deposit plus the first deposit plus the interest on the first deposit.) After the third deposit, the balance is $1000 + [1000 + 1000(0.085)] + [1000 + 1000(1.085)](0.085) = 1000 + [1000 + 1000(1.085)](1.085) = 1000 + 1000(1.085) + 1000(1.085)^2$. The trend is as follows: after the $n$th deposit, the balance is $1000 + 1000(1.085) + \ldots + 1000(1.085)^n$. Thus, after the 20th deposit, the balance is $1000 \frac{1 - 1.085^{20}}{1 - 1.085} \approx 48377.01$. Since we deposited $1000 per year for 20 years, we put in $20,000 and earned 28377.01 in interest.

2. The balance after the first deposit is $2000. After the second deposit, it is $2000 + 2000e^{0.07}$. (Remember how continuous compounding works?) After the third deposit, we have $2000 + (2000 + 2000e^{0.07})e^{0.07} = 2000 + 2000e^{0.07} + 2000(e^{0.07})^2$. After the $n$th deposit, we have $2000 + 2000e^{0.07} + \ldots + 2000(e^{0.07})^{n-1} = 2000 \frac{1 - (e^{0.07})^n}{1 - e^{0.07}}$.

Right after the fifth deposit, the account balance is $2000 \frac{1 - (e^{0.07})^5}{1 - e^{0.07}} \approx 11559.18$. Right before this deposit, the balance was $2000 less, or $9559.18.

8. (a) The total amount spent would be $5(0.8) + 5(0.8)^2 + \ldots = \frac{5(0.8)}{1 - 0.8} = 20$ billion dollars.

(b) The total amount spent would be $5(0.9) + 5(0.9)^2 + \ldots = \frac{5(0.9)}{1 - 0.9} = 45$ billion dollars. Lots more!

11. The sum would be $1 + 2 + 2^2 + \ldots + 2^n = \frac{1 - 2^{n+1}}{1 - 2} = 2^{n+1} - 1$ for $n$ days of work.

(a) With $n = 7$, this is $2^{7+1} - 1 = 256 - 1 = 255$ cents.

(b) With $n = 14$, we get $2^{14+1} - 1 = 32767$ cents, or $327.67$.

(c) Now $n = 21$, giving $2^{21+1} - 1 = 4194303$ cents, or $41943.03$.

(d) Finally, with $n = 28$, we get $2^{28+1} - 1 = 536870911$ cents, or $5,368,709.11$. Not bad for a month.

12. Each year, the employee’s salary is multiplied by 1.04, so at the start of the 11th year, her salary is $30000(1.04)^{10} = 44,407.33$ (There were only 10 raises.) Her total earnings are $30000 + 30000(1.04) + \ldots + 30000(1.04)^9 = 30000 \frac{1 - 1.04^{10}}{1 - 1.04} \approx 360,183.21$. 

Solutions to Homework Assignment 34